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GENERALIZED FACTOR ANALYSIS

PART II

APPLICATIONS

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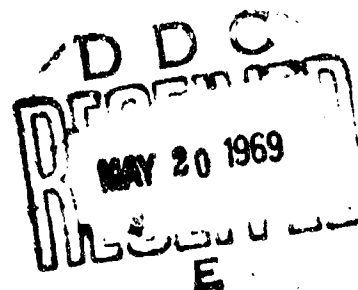
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## GENERALIZED FACTOR ANALYSIS

### Part I Rationale

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## CHAPTER 11

### THE DATA SETS

The methods developed in the previous chapters were applied to twelve different sets of data. These will now be described.

#### 11.1 Primary Mental Abilities

Thurstone and Thurstone (1941) administered 60 tests to 710 eighth grade students. The intercorrelations analyzed were taken from nine of these tests. The first three of these were verbal tests, the next three spatial, and the last three numerical, as follows:

1. Sentences
2. Vocabulary
3. Completion
4. Flags
5. Figures
6. Cards
7. Addition
8. Multiplication
9. Three higher

#### 11.2 Twenty-Four Psychological Tests

This set of correlations comes from a battery of twenty-four psychological tests given to 145 seventh and eighth grade school children in a suburb of Chicago. The initial data were gathered by Holzinger and Swineford (1939). The data have subsequently been analyzed by a number of investigators including Holzinger and Harmon (1941), Kaiser (1958), Neuhaus and Wrigley (1954), Harmon (1967), and others, so that the characteristics of the data have come to be well known. The tests are identified as follows:

- |                            |                              |
|----------------------------|------------------------------|
| 1. Visual perception       | 13. Straight-curved capitals |
| 2. Cubes                   | 14. Word recognition         |
| 3. Paper form board        | 15. Number recognition       |
| 4. Flags                   | 16. Figure recognition       |
| 5. General information     | 17. Object-number            |
| 6. Paragraph comprehension | 18. Number-figure            |
| 7. Sentence completion     | 19. Figure-word              |
| 8. Word classification     | 20. Deduction                |
| 9. Word meaning            | 21. Numerical puzzles        |
| 10. Addition               | 22. Problem reasoning        |
| 11. Code                   | 23. Series completion        |
| 12. Counting               | 24. Arithmetic problems      |

### 11.3 Thirty-Three Variable Speed Study

These data are from a study by Lord (1956) designed to investigate the speed factor. Tests were administered to 649 students in the entering class at the United States Naval Academy at Annapolis. The tests were designed to measure verbal, spatial, and arithmetic reasoning ability. In each area, seven tests were administered. One was the regular admissions examination denoted by (A). The remaining six were short experimental tests parallel in content but different in degree of speededness. Two designated (L) involved virtually no speed, one was moderately speeded (M), and the remaining three (S) were highly speeded. Six reference factor tests designated by (R) were also included. In addition, grades in six areas designated (G) were included as variables. The 33 variables are as follows:

- |                              |   |
|------------------------------|---|
| 1. Word fluency (R)          | 18. Arithmetic reasoning (L)                            |
| 2. Verbal (A)                | 19. Arithmetic reasoning (M)                            |
| 3. Vocabulary (L)            | 20. Arithmetic reasoning (S)                            |
| 4. Vocabulary (L)            | 21. Arithmetic reasoning (S)                            |
| 5. Vocabulary (M)            | 22. Arithmetic reasoning (S)                            |
| 6. Vocabulary (S)            | 23. Number speed (R)                                    |
| 7. Vocabulary (S)            | 24. Number speed (R)                                    |
| 8. Vocabulary (S)            | 25. Cancellation (R)                                    |
| 9. Spatial relations (A)     | 26. Picture discrimination (R)                          |
| 10. Intersections (L)        | 27. Number checking (R)                                 |
| 11. Intersections (L)        | 28. English (G)   |
| 12. Intersections (M)        | 29. Foreign language (G)                                |
| 13. Intersections (S)        | 30. Engineering drawing and<br>descriptive geometry (G) |
| 14. Intersections (S)        | 31. Chemistry (G)                                       |
| 15. Intersections (S)        | 32. Mathematics (G)                                     |
| 16. Mathematics (A)          | 33. Conduct   |
| 17. Arithmetic reasoning (L) |   |

#### 11.4 Thurstone Twenty-Variable Box Problem

These data are from the classical study by Thurstone (1947) designed to illustrate the principle of simple structure. Measurements of a random collection of thirty boxes were made. The three dimensions X, Y, and Z were recorded for each box. A list of 26 arbitrary score functions was then prepared. Twenty of these functions were included as variables in our analysis. These are as follows:

- |            |                              |
|------------|------------------------------|
| 1. $X$     | 11. $Y^2Z$                   |
| 2. $Y$     | 12. $YZ^2$                   |
| 3. $Z$     | 13. $2X + 2Y$                |
| 4. $XY$    | 14. $2X + 2Z$                |
| 5. $XZ$    | 15. $2Y + 2Z$                |
| 6. $YZ$    | 16. $\sqrt{X^2 + Y^2}$       |
| 7. $X^2Y$  | 17. $\sqrt{X^2 + Z^2}$       |
| 8. $XY^2$  | 18. $\sqrt{Y^2 + Z^2}$       |
| 9. $X^2Z$  | 19. $XYZ$                    |
| 10. $XZ^2$ | 20. $\sqrt{X^2 + Y^2 + Z^2}$ |

#### 11.5 Eight-Variable Body Type Measures

These data are from a study of eight physical variables by Mullen (1939).

The data have been used for illustrative purposes by Harmon (1967) and by Kaiser and Caffrey (1965). The variables are as follows:

- |                        |                            |
|------------------------|----------------------------|
| 1. Height              | 5. Weight                  |
| 2. Arm span            | 6. Bitrochanteric diameter |
| 3. Length of forearm   | 7. Chest girth             |
| 4. Length of lower leg | 8. Chest width             |

#### 11.6 Twelve-Variable Anthropometric Measures

These data are from a factor analysis by Hammond (1942) involving twelve body measurements on adult men. Hammond attempted to interpret the resulting factor matrix without rotation of axes. Later Thurstone (1946) reanalyzed the data rotating to simple structure. The variables are as follows:

- |                     |                  |
|---------------------|------------------|
| 1. Stature          | 7. Chest depth   |
| 2. Sitting height   | 8. Head length   |
| 3. Shoulder breadth | 9. Head breadth  |
| 4. Hip breadth      | 10. Head height  |
| 5. Span             | 11. Hand length  |
| 6. Chest Breadth    | 12. Hand breadth |

#### 11.7 Fifteen Variables from Hemmerle

These data are from a study by Hemmerle (1965) designed to illustrate a method for obtaining maximum likelihood estimates of factor loadings and communalities using an iterative computer procedure. Later the data were reanalyzed by methods developed by Jöreskog (1967) and by Horst (1968b). This data set was included because of the divergent results obtained by the several investigators. Hemmerle does not indicate the source of the data, the number of cases, nor the nature of the variables.

#### 11.8 Seventeen-Variable Data from Bechtold--Sample 1

These data are from a study by Bechtold (1961) designed to investigate the factor analysis stability hypothesis. The data are a portion of those originally collected by Thurstone and Thurstone (1941). The study included seventeen variables from a sample of 212 cases. The first two variables were designed to measure memory (M), the next three verbal ability (V), and successive sets of three measure word fluency (W), spatial ability (S), number ability (N), and reasoning ability (R). The seventeen variables were given designations as follows:

- |                          |                           |
|--------------------------|---------------------------|
| 1. First names (M)       | 10. Figures (S)           |
| 2. Word-number (M)       | 11. Cards (S)             |
| 3. Sentences (V)         | 12. Addition (N)          |
| 4. Vocabulary (V)        | 13. Multiplication (N)    |
| 5. Completion (V)        | 14. Three higher (N)      |
| 6. First letters (W)     | 15. Letter series (R)     |
| 7. Four-letter words (W) | 16. Pedigrees (R)         |
| 8. Suffixes (W)          | 17. Letter groupings: (R) |
| 9. Flags (S)             |                           |

#### 11.9 Seventeen-Variable Data from Bechtold--Sample 2

These data are from the same study by Bechtold (1961) as those in Section 11.8. The variables are the same as in that data set but the cases are a separate sample of 213 cases. The two samples of cases were formed by assigning each of 425 cases alternately to one or the other of two groups after the cases were thoroughly randomized.

#### 11.10 Nine-Variable Synthetic Data

The correlation matrix for this data set was derived from a configuration of points constructed so as to provide a severe test for the simple structure transformation procedure described in Chapter 9. A right spherical triangle was constructed on the surface of a sphere of unit radius. A point was located on each side of the triangle midway between the two vertices, or 45 degrees from each of the two vertices. Two more points were located on each side of the spherical triangle, one each midway between a vertex and the mid point, or 22.5 degrees from a vertex and the mid point. Thus the three points on the 90 degree arc of the great circle constituting a side of the triangle divided the side into four equal arcs of 22.5 degrees each. The cosines of the angular distances between all pairs of the nine points were calculated to obtain a correlation matrix. The cosines



of the angular distances of each of the nine points with each of the three vertices of the right spherical triangle were also calculated. These values are the simple structure factor loadings of the variables (points). An adequate method of analysis of the correlation matrix including simple structure transformation should recover the synthetically constructed simple structure factor loadings.

#### 11.11 Reading Comprehension Factors

These data are from a study by Davis (1944) designed to investigate the primary factors of reading comprehension. Tests were designed to measure nine different reading skills. The correlations are based on scores of 421 college freshman. The tests were as follows:

1. Knowledge of word meaning
2. Ability to select the appropriate meaning for a word or phrase in the light of its particular contextual setting
3. Ability to follow the organization of a passage and to identify antecedents and references to it
4. Ability to select the main thought in a passage
5. Ability to answer questions that are specifically answered in a passage
6. Ability to answer questions that are answered in a passage but not in words in which the question is asked
7. Ability to draw inferences from a passage about its contents
8. Ability to recognize the literary devices used in a passage and to determine its tone and mode
9. Ability to determine a writer's purpose, intent, and point of view, i.e., to draw inferences about a writer.

### 11.12 The Heywood Case

These data are from a five-variable synthetic example from Thomson (1950).

The correlation matrix was constructed so that every tetrad difference is exactly zero, but the g factor saturation for one of the tests is greater than unity. However, the matrix is positive definite. This example was included to test the behavior of various scaling and loss function parameters described in Chapter 8 for the Heywood case. The loadings of the variables for the g factor were chosen as follows:

1. 1.05
2. .9
3. .8
4. .7
5. .6

## CHAPTER 12

### EXPERIMENTAL RESULTS

In this chapter we shall merely present the numerical results of the analyses for the twelve data sets. In Chapter 13 we shall discuss some of the more interesting of these results. At the end of this chapter the results are presented successively for each of the twelve data sets. For each data set six separate sets of analyses are presented. For the first group of three of these analyses the loss function parameter  $P_W = 1$  was used and for the second group of three this parameter was  $P_W = 0$ . Within each group, the first set is for the scaling parameter  $p = 0$ , the second for  $p = .5$ , and the third for  $p = 1.0$ . The format for all sets of data is identical. It consists of a first line, a second line or sequence of lines, a third block of lines, and a final line. These we shall now interpret.

#### 12.1 The First Line

The first line has three successive groups of numbers. The first group consists of six integers. The second consists of three figures. The third group has figures equal in number to the number of roots  $m$  of the correlation matrix greater than unity.

The six integers in the first group are as follows:

- (1) The first integer is simply the arbitrary serial order of the data set.
- (2) The second integer is the order of the correlation matrix or the number of variables  $n$ .
- (3) The third integer is the number of factors solved for. This is the number of roots of the correlation matrix greater than unity. It is the same as the number of figures in the last group of the first line.
- (4) The fourth integer is a code for the loss function parameter  $P_W$ . For  $P_W = 1$  the integer is 1 and for  $P_W = 0$  the integer is 2. Thus the integer 1 means that the loss function includes only the residual covariance elements, while the integer 2 means that it includes both the unit weighted residual variance and

covariance elements. It is possible, of course, to have  $P_W$  take any value between unity and zero but only the two extremes were used for all twelve data sets.

(5) The fifth integer is a code for the scaling function parameter  $p$ . For  $p = 0$  the integer is 1, for  $p = .5$  it is 2, and for  $p = 1.0$  it is 3. Thus the integer 1 means that the scaling function is the square root reciprocal of the residual variance, the number 2 means that it is the square root residual of the total variance, and the number 3 means that it is the square root residual of the estimated or common variance. It is possible, of course, to let  $p$  take any value between zero and unity but only the three values indicated above were used for the twelve data sets.

(6) The sixth and final integer in this group is a code to indicate the row scaling treatment of the factor loading matrix prior to the simple structure procedures of Chapter 9. The integer 1 indicates that the factor loading matrix was normalized by rows prior to simple structure transformation; if it was not, the integer 2 is used to so indicate. The computer program provides for both options but in this study only the normalizing option was used for all sets of data. Hence for each of the six analyses for all twelve data sets, the last integer in the first group of six in the first row is always 1.

The three figures in the second group are as follows:

(1) The first figure in this group is the ratio of the sum of squares of the first  $m$  roots of the matrix for specified scaling and loss function parameters to the sum of squares of all the roots of this matrix. This is the criterion  $\phi$  developed in Chapter 8 which it is desired to maximize. The maximum value it can attain is unity.

(2) The second figure in this group is the number of iterations required to reach the tolerance limits for the equality of two successive iterations for  $\phi$  or the iteration limit, whichever is reached first.

(3) This figure is the time in minutes taken for the required number of iterations.

In the third group of  $m$  figures,  $m$  is the number of roots of the original correlation matrix greater than unity. The  $m$  figures in this group are the  $m$  largest roots of the matrix for the specified scaling and loss function parameters.

## 12.2 The Second Line or Sequence of Lines

The second line or sequence of lines consists of four numbers to a line. The numbers all have to do with the simple structure transformation described in Chapter 9. Since data sets 11 and 12 have only a single factor, no transformations were required, hence no lines of four numbers appear for these data sets. For data sets 1 through 9, no more than one or two lines are given. For data set 10, 20 lines of four numbers each are given. The four numbers of each line have the following interpretations:

(1) The first number is the quantity  $\text{tr}(D\Delta)$  where  $D$  and  $\Delta$  are defined in Eqs. 9.21 and 9.34. This value is calculated at each iteration for the simple structure transformation matrix. When these values for two successive iterations are within the specified tolerance limit, the iterations cease. An iteration limit is also specified in the computer program beyond which iterations cease even though the tolerance limit is not yet reached.

(2) The second number is the simple structure criterion  $\Psi$  given in Eq. 9.70. The maximum value this criterion can attain is unity.

(3) The third number is the number of sets of iterations taken to calculate the simple structure factor loading matrix for a given positive integer  $W$  used to calculate  $F$  in Eq. 9.58. This integer is 1 for the first set of iterations. If the number of negative factor loadings in any column is less than the number of factors, the computations cease. If not, the integer  $W$  is increased by 1 and a second set of iterations for the simple structure factor loading matrix occurs.

This procedure continues until at least one column of the simple structure factor loading matrix has fewer negatives than the number of factors or columns in the matrix. When this occurs, the simple structure matrix from the preceding set of iterations is taken as the final simple structure factor matrix, except for  $W = 1$ , in which case the corresponding simple structure factor loading matrix is accepted. A limit is put on the number of successive sets of iterations. If this limit is reached before the number of negative values in any column of the simple structure factor loading matrix is less than the number of factors, the successive sets of iterations cease.

(4) This number is the integer  $W$ . It indicates the number of sets of iterations calculated and is therefore the number of the line in the sequence of lines. The integer  $W$  serves as the argument for the power function in the numerator terms of the criterion function  $\Psi$  given by Eq. 9.70 and defined in more detail in preceding parts of Chapter 9.

### 12.3 The Factor Loading Lines

A set of  $n$  lines, where  $n$  is the number of variables, is given for each of the data sets. The columns in this set of lines are as follows:

- (1) The first column gives simply the line numbers which indicate, of course, also the arbitrary serial numbers of the variables or tests described in Chapter 12.
- (2) The second column has a 1 if the variable retains its original sign and -1 if its sign is reversed as discussed in Chapter 6, Section 4.
- (3) The third column gives the communalities of the variables as calculated from the factor loading matrix calculated by the methods of Chapter 8.
- (4) The fourth column gives the specificities corresponding to the communalities in the second column. The sum of corresponding elements of the two columns is therefore unity.

(5) The next block of  $m$  columns gives the factor loading matrix for  $m$  factors calculated by the methods of Chapter 8.

(6) The last block of  $m$  columns for data sets 1 through 10 gives the simple structure factor loading matrix calculated by the methods of Chapter 9. For data sets 11 and 12, this block of columns is omitted since only one factor loading vector was calculated for each set.

#### 12.4 The Last Line

The last line for data sets 1 through 10 consists of  $m$  figures, where  $m$  is the number of factors solved for. Each value is the corresponding element of the  $\Delta$  diagonal matrix defined in Eq. 9.34. These elements are the ratios whose average is given by  $\bar{Y}$  in Eq. 9.70. It is the average of these ratios which is the second number in the second row or sequence of rows described in Section 12.2. This is the simple structure criterion we seek to maximize. The maximum value any one of these numbers can take is unity.

This final row of figures is not given for data sets 10 and 11, since only one factor vector was calculated for each.

## DATA SET 1

14.634													6.171			3.232					
1.068 0.050 11.000 1.000																					
1	1	0.926	0.174	0.942	-0.257	-0.131	0.823	-0.035	0.014	0.910	-0.019	0.050									
2	1	0.834	0.166	0.877	-0.233	-0.101	0.762	0.163	-0.017	0.800	0.724	-0.117									
3	1	0.756	0.244	0.846	-0.056	-0.142	-0.025	0.840	-0.014	0.734	0.807	0.303									
4	1	0.595	0.405	0.826	0.697	-0.060	-0.005	0.059	0.698	-0.015	-0.036	0.765									
5	1	0.722	0.278	0.250	0.788	-0.193	0.115	0.219	0.511	0.923	-0.035	0.013									
6	1	0.635	0.315	0.317	0.745	-0.182	0.810	-0.020	0.047	0.762	0.171	-0.014									
7	1	0.617	0.343	0.488	0.173	0.543	0.001	0.725	0.122	-0.024	0.838	-0.016									
8	1	0.701	0.293	0.402	0.105	0.663	0.030	0.807	0.006	-0.005	0.060	0.698									
9	1	0.500	0.500	0.542	0.252	0.370	-0.016	-0.035	0.764	0.115	0.223	0.512									
0.075 0.051 0.563													1.455			1.763			1.016		
1.068 0.050 14.000 1.000																					
1	1	0.825	0.175	0.777	-0.465	-0.284	0.923	-0.035	0.013	0.810	-0.020	0.047									
2	1	0.833	0.167	0.751	-0.450	-0.256	0.762	0.171	-0.014	0.800	0.724	-0.117									
3	1	0.758	0.242	0.764	-0.263	-0.376	0.001	0.725	0.122	-0.024	0.838	-0.016									
4	1	0.590	0.401	0.505	0.584	-0.050	0.030	0.807	0.006	-0.005	0.060	0.698									
5	1	0.713	0.287	0.434	0.706	-0.177	0.115	0.219	0.511	0.923	-0.035	0.013									
6	1	0.634	0.316	0.427	0.540	-0.174	0.810	-0.020	0.047	0.762	0.171	-0.014									
7	1	0.617	0.343	0.503	-0.028	0.515	0.001	0.725	0.122	-0.024	0.838	-0.016									
8	1	0.698	0.292	0.583	-0.109	0.530	0.030	0.807	0.006	-0.005	0.060	0.698									
9	1	0.500	0.500	0.641	0.004	0.297	-0.016	-0.035	0.764	0.115	0.223	0.512									
0.075 0.051 0.563													5.037			2.481			1.482		
1.068 0.050 18.000 1.000																					
1	1	0.825	0.175	0.653	0.502	-0.383	0.823	-0.034	0.013	0.910	-0.021	0.046									
2	1	0.832	0.160	0.680	0.433	-0.355	0.762	0.172	-0.015	0.800	0.727	0.125									
3	1	0.750	0.243	0.703	0.293	-0.410	0.001	0.727	0.125	-0.024	0.837	-0.017									
4	1	0.693	0.307	0.558	-0.531	-0.050	0.030	0.806	0.007	-0.005	0.061	0.697									
5	1	0.715	0.285	0.480	-0.675	-0.154	0.115	0.223	0.512	0.923	-0.035	0.013									
6	1	0.642	0.314	0.500	-0.514	-0.153	0.810	-0.021	0.046	0.762	0.172	-0.015									
7	1	0.616	0.344	0.624	0.131	0.458	0.001	0.727	0.125	-0.024	0.837	-0.017									
8	1	0.697	0.293	0.611	0.217	0.525	0.030	0.806	0.007	-0.005	0.061	0.697									
9	1	0.500	0.500	0.566	0.073	0.266	-0.016	-0.035	0.763	0.115	0.223	0.512									
0.075 0.051 0.563													1.574			0.561					





## DATA SET 2

2	24	5	1	1	1	0.500	0.000	1.000	0.167	19.172	5.460	3.101	1.930	1.309
0.934	0.500	0.000	1.000											
1	1.	0.549	0.651	0.561	0.025	-0.452	0.128	0.026	0.013	0.237	0.536	0.053	0.035	0.035
2	1.	0.229	0.770	0.345	-0.032	-0.293	0.036	0.123	0.040	0.035	0.389	0.010	0.010	0.086
3	1.	0.361	0.639	0.377	-0.126	-0.435	0.109	0.020	0.024	0.156	0.453	0.031	-0.076	-0.076
4	1.	0.553	0.447	0.462	-0.096	-0.300	0.109	0.162	0.142	0.081	0.457	-0.054	0.097	0.097
5	1.	0.644	0.356	0.726	-0.255	0.317	0.063	-0.013	0.660	0.276	0.014	0.110	0.100	0.100
6	1.	0.712	0.288	0.722	-0.321	0.152	0.002	-0.142	0.682	0.340	-0.015	0.131	-0.065	-0.065
7	1.	0.772	0.228	0.722	-0.355	0.235	0.128	0.014	0.728	0.291	0.014	-0.077	0.050	0.050
8	1.	0.816	0.184	0.689	-0.153	0.327	0.100	0.065	0.477	0.225	0.200	-0.024	0.144	0.144
9	1.	0.712	0.288	0.726	-0.426	0.169	-0.020	-0.029	0.727	0.241	0.037	0.081	0.003	0.003
10	1.	0.743	0.257	0.512	0.589	0.390	-0.041	0.126	0.149	0.059	-0.093	0.009	0.644	0.644
11	1.	0.804	0.196	0.571	0.374	0.140	-0.070	-0.256	0.043	0.434	-0.045	0.364	0.102	0.102
12	1.	0.856	0.144	0.670	0.567	0.322	0.117	0.132	-0.052	0.124	0.159	-0.051	0.490	0.490
13	1.	0.757	0.243	0.629	0.351	-0.193	0.167	-0.214	0.006	0.586	0.225	-0.023	0.064	0.064
14	1.	0.755	0.245	0.603	0.011	-0.069	-0.363	-0.254	0.116	0.131	-0.002	0.506	-0.021	-0.021
15	1.	0.826	0.174	0.354	0.023	-0.104	-0.364	-0.129	0.057	0.074	0.359	0.445	0.053	0.053
16	1.	0.857	0.143	0.454	0.036	-0.145	-0.294	-0.111	-0.051	0.074	0.359	0.445	0.010	0.010
17	1.	0.890	0.110	0.437	0.134	-0.033	-0.404	-0.121	0.076	0.023	0.070	0.471	0.162	0.162
18	1.	0.905	0.095	0.471	0.265	-0.211	-0.260	-0.028	-0.064	-0.003	0.309	0.323	0.276	0.276
19	1.	0.879	0.121	0.417	0.063	-0.151	-0.179	-0.055	0.066	0.084	0.199	0.271	0.096	0.096
20	1.	0.847	0.153	0.502	-0.152	-0.173	-0.161	0.225	0.304	-0.071	0.410	0.215	0.236	0.236
21	1.	0.817	0.181	0.574	0.235	-0.149	-0.024	0.181	0.084	0.036	0.370	0.055	0.377	0.377
22	1.	0.813	0.187	0.585	-0.126	-0.164	-0.100	0.134	0.281	0.030	0.359	0.114	0.167	0.167
23	1.	0.854	0.146	0.584	-0.052	-0.211	-0.033	0.251	0.271	0.003	0.484	0.027	0.232	0.232
24	1.	0.824	0.176	0.659	0.184	0.139	-0.138	0.138	0.311	0.037	0.145	0.121	0.436	0.436

0.577 0.530 0.414 0.446 0.591



## DATA SET 2 (cont.)

	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441	442	443	444	445	446	447	448	449	450	451	452	453	454	455	456	457	458	459	460	461	462	463	464	465	466	467	468	469	470	471	472	473	474	475	476	477	478	479	480	481	482	483	484	485	486	487	488	489	490	491	492	493	494	495	496	497	498	499	500	501	502	503	504	505	506	507	508	509	510	511	512	513	514	515	516	517	518	519	520	521	522	523	524	525	526	527	528	529	530	531	532	533	534	535	536	537	538	539	540	541	542	543	544	545	546	547	548	549	550	551	552	553	554	555	556	557	558	559	560	561	562	563	564	565	566	567	568	569	570	571	572	573	574	575	576	577	578	579	580	581	582	583	584	585	586	587	588	589	590	591	592	593	594	595	596	597	598	599	600	601	602	603	604	605	606	607	608	609	610	611	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	632	633	634	635	636	637	638	639	640	641	642	643	644	645	646	647	648	649	650	651	652	653	654	655	656	657	658	659	660	661	662	663	664	665	666	667	668	669	670	671	672	673	674	675	676	677	678	679	680	681	682	683	684	685	686	687	688	689	690	691	692	693	694	695	696	697	698	699	700	701	702	703	704	705	706	707	708	709	710	711	712	713	714	715	716	717	718	719	720	721	722	723	724	725	726	727	728	729	730	731	732	733	734	735	736	737	738	739	740	741	742	743	744	745	746	747	748	749	750	751	752	753	754	755	756	757	758	759	760	761	762	763	764	765	766	767	768	769	770	771	772	773	774	775	776	777	778	779	780	781	782	783	784	785	786	787	788	789	790	791	792	793	794	795	796	797	798	799	800	801	802	803	804	805	806	807	808	809	810	811	812	813	814	815	816	817	818	819	820	821	822	823	824	825	826	827	828	829	830	831	832	833	834	835	836	837	838	839	840	841	842	843	844	845	846	847	848	849	850	851	852	853	854	855	856	857	858	859	860	861	862	863	864	865	866	867	868	869	870	871	872	873	874	875	876	877	878	879	880	881	882	883	884	885	886	887	888	889	890	891	892	893	894	895	896	897	898	899	900	901	902	903	904	905	906	907	908	909	910	911	912	913	914	915	916	917	918	919	920	921	922	923	924	925	926	927	928	929	930	931	932	933	934	935	936	937	938	939	940	941	942	943	944	945	946	947	948	949	950	951	952	953	954	955	956	957	958	959	960	961	962	963	964	965	966	967	968	969	970	971	972	973	974	975	976	977	978	979	980	981	982	983	984	985	986	987	988	989	990	991	992	993	994	995	996	997	998	999	1000	1001	1002	1003	1004	1005	1006	1007	1008	1009	1010	1011	1012	1013	1014	1015	1016	1017	1018	1019	1020	1021	1022	1023	1024	1025	1026	1027	1028	1029	1030	1031	1032	1033	1034	1035	1036	1037	1038	1039	1040	1041	1042	1043	1044	1045	1046	1047	1048	1049	1050	1051	1052	1053	1054	1055	1056	1057	1058	1059	1060	1061	1062	1063	1064	1065	1066	1067	1068	1069	1070	1071	1072	1073	1074	1075	1076	1077	1078	1079	1080	1081	1082	1083	1084	1085	1086	1087	1088	1089	1090	1091	1092	1093	1094	1095	1096	1097	1098	1099	1100	1101	1102	1103	1104	1105	1106	1107	1108	1109	1110	1111	1112	1113	1114	1115	1116	1117	1118	1119	1120	1121	1122	1123	1124	1125	1126	1127	1128	1129	1130	1131	1132	1133	1134	1135	1136	1137	1138	1139	1140	1141	1142	1143	1144	1145	1146	1147	1148	1149	1150	1151	1152	1153	1154	1155	1156	1157	1158	1159	1160	1161	1162	1163	1164	1165	1166	1167	1168	1169	1170	1171	1172	1173	1174	1175	1176	1177	1178	1179	1180	1181	1182	1183	1184	1185	1186	1187	1188	1189	1190	1191	1192	1193	1194	1195	1196	1197	1198	1199	1200	1201	1202	1203	1204	1205	1206	1207	1208	1209	1210	1211	1212	1213	1214	1215	1216	1217	1218	1219	1220	1221	122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[illegible]

3	33	6	1	1	1	0.999	20.000	0.150	36.484	19.453	9.413	4.443	3.601	1.102
0.752	0.536	60.000	1.000											
0.726	0.509	53.000	2.000											
1	1	0.139	0.912	0.343	-0.145	0.100	0.068	0.168	-0.082		0.202	0.063	0.248	0.239
2	1	0.799	0.201	0.676	-0.472	-0.129	0.005	-0.171	-0.082		0.394	0.002	0.597	0.093
3	1	0.653	0.362	0.563	-0.479	-0.249	0.021	-0.105	-0.131		0.420	-0.019	0.537	-0.054
4	1	0.675	0.225	0.630	-0.430	-0.264	0.085	-0.138	-0.037		0.440	0.016	0.559	0.000
5	1	0.754	0.246	0.627	-0.525	-0.231	0.024	-0.042	-0.158		0.453	-0.024	0.543	0.045
6	1	0.701	0.296	0.634	-0.431	-0.153	0.060	0.120	0.097		0.635	0.051	0.411	-0.005
7	1	0.874	0.126	0.707	-0.563	-0.166	0.053	0.156	0.094		0.718	-0.017	0.479	0.131
8	1	0.827	0.163	0.603	-0.530	-0.116	0.012	0.176	0.174		0.741	-0.026	0.386	0.007
9	1	0.605	0.554	0.476	0.384	-0.147	-0.105	0.014	-0.037		0.321	0.492	-0.012	0.131
10	1	0.657	0.343	0.555	0.542	-0.211	0.025	-0.014	-0.079		-0.045	0.684	0.068	0.033
11	1	0.728	0.272	0.530	0.614	-0.250	0.030	0.026	-0.190		-0.051	0.753	0.036	-0.035
12	1	0.767	0.233	0.576	0.601	-0.272	0.003	-0.015	0.099		0.013	0.733	0.008	0.015
13	1	0.759	0.241	0.579	0.591	-0.251	0.001	0.102	-0.011		0.047	0.768	-0.032	-0.033
14	1	0.805	0.125	0.561	0.627	-0.248	0.015	0.116	-0.011		0.035	0.807	-0.035	0.032
15	1	0.814	0.186	0.593	0.637	-0.222	-0.053	0.118	-0.031		0.012	0.799	-0.062	0.053
16	1	0.645	0.355	0.620	0.615	-0.277	-0.364	-0.223	-0.033		-0.008	-0.003	0.077	0.613
17	1	0.521	0.459	0.589	0.095	0.195	-0.216	-0.297	0.045		0.305	0.048	0.103	-0.464
18	1	0.562	0.453	0.539	0.099	0.255	-0.221	-0.286	-0.002		-0.022	0.046	0.115	0.494
19	1	0.620	0.330	0.617	0.052	0.216	-0.356	-0.243	0.055		0.043	0.024	0.023	-0.576
20	1	0.670	0.230	0.537	0.349	0.241	-0.401	-0.102	0.109		0.106	0.041	-0.045	0.502
21	1	0.599	0.411	0.635	0.090	0.247	-0.303	-0.123	0.104		0.103	0.079	0.010	0.514
22	1	0.597	0.403	0.613	-0.013	0.233	-0.362	-0.087	0.050		0.104	0.011	0.019	0.562
23	1	0.625	0.376	0.238	-0.113	0.539	-0.190	0.420	-0.226		-0.010	-0.043	0.043	0.363
24	1	0.664	0.255	0.274	-0.037	0.567	-0.301	0.343	-0.179		-0.039	-0.069	-0.323	0.688
25	1	0.627	0.693	0.259	-0.028	0.268	-0.002	0.421	-0.005		0.199	0.124	0.022	0.355
26	1	0.363	0.641	0.355	0.004	0.093	-0.036	0.452	0.009		0.241	0.323	-0.010	0.021
27	1	0.425	0.575	0.120	-0.040	0.423	-0.031	0.473	-0.065		0.073	0.021	-0.023	0.121
28	1	0.566	0.434	0.580	-0.354	0.109	0.270	-0.102	-0.093		0.310	-0.076	0.616	-0.012
29	1	0.272	0.623	0.358	-0.029	0.325	0.238	-0.034	-0.113		0.322	0.005	0.436	-0.024
30	1	0.652	0.248	0.311	0.423	0.032	0.276	0.064	0.032		0.112	0.445	0.202	0.119
31	1	0.816	0.134	0.601	0.177	0.274	0.416	-0.146	0.042		0.039	0.037	0.423	0.053
32	1	0.815	0.165	0.572	0.215	0.533	0.312	-0.055	-0.006		-0.039	0.101	0.350	0.039
33	1	0.052	0.543	0.056	0.054	0.149	0.047	-0.053	0.117		-0.014	-0.030	-0.030	0.196
0.516	0.052	0.516	0.594	0.521										0.232









## MPT 3 (cont.)

	3	33	5	7	2	1	0.500	4.000	6.000	10.265	4.774	3.145	1.982	1.657	1.024
0.740	1.547	60.003	1.000												
0.537	0.537	26.003	1.000												
1	1.	1.642	1.558	0.356	0.204	0.097	0.281	-0.019	0.428	0.294	0.077	0.301	-0.020	0.392	-0.097
2	1.	0.787	0.213	0.523	0.501	-0.393	-0.019	-0.013	-0.016	0.770	-0.002	-0.074	0.068	0.348	0.435
3	1.	0.700	0.300	0.532	0.515	-0.346	0.015	-0.044	0.117	0.755	-0.025	-0.054	0.021	-0.012	0.387
4	1.	1.732	0.279	0.582	0.463	-0.398	0.032	0.059	-0.027	0.756	-0.017	-0.057	-0.015	0.065	0.459
5	1.	0.725	0.335	0.567	0.557	-0.365	0.044	-0.037	-0.027	0.799	-0.026	-0.019	0.019	-0.018	0.451
6	1.	0.730	0.270	0.633	0.460	-0.243	0.184	-0.095	-0.036	0.765	0.115	0.173	0.015	0.002	0.484
7	1.	0.469	0.151	0.627	0.581	-0.241	0.158	-0.037	-0.022	0.839	0.037	0.196	0.013	-0.019	0.513
8	1.	0.792	0.211	0.631	0.553	-0.188	0.180	-0.037	-0.022	0.731	0.042	0.217	0.047	-0.023	0.504
9	1.	1.664	0.506	0.516	-0.411	-0.143	0.016	-0.139	-0.139	0.028	0.570	0.035	0.129	0.259	0.394
10	1.	0.704	0.236	0.582	-0.544	-0.222	0.089	-0.006	0.080	-0.005	0.693	-0.044	0.014	0.493	-0.024
11	1.	1.764	0.236	0.554	-0.619	-0.236	0.127	-0.023	0.042	-0.030	0.764	-0.034	-0.023	0.470	-0.023
12	1.	0.756	0.204	0.594	-0.604	-0.270	0.064	-0.031	-0.031	-0.010	0.753	-0.076	0.013	0.452	0.026
13	1.	1.782	0.213	0.595	-0.583	-0.223	0.154	-0.090	0.032	0.014	0.785	0.033	-0.004	0.439	-0.011
14	1.	0.805	0.195	0.572	-0.623	-0.230	0.146	-0.107	-0.031	-0.010	0.803	0.020	-0.010	0.413	-0.010
15	1.	0.811	0.120	0.602	-0.626	-0.170	0.113	-0.115	-0.004	-0.043	0.792	0.054	0.055	0.442	-0.021
16	1.	0.695	0.215	0.683	0.046	0.212	-0.301	-0.136	0.075	0.057	0.011	0.084	0.674	0.359	0.033
17	1.	0.692	0.608	0.648	-0.030	0.026	-0.207	0.026	0.059	0.031	0.040	-0.085	0.555	0.459	0.024
18	1.	0.622	0.408	0.650	-0.037	0.153	-0.384	0.021	0.147	-0.003	0.021	-0.041	0.582	0.510	-0.051
19	1.	0.565	0.235	0.670	0.004	0.151	-0.403	-0.121	0.035	0.055	0.040	0.025	0.543	0.356	0.030
20	1.	0.660	0.303	0.605	0.015	0.249	-0.346	-0.175	-0.044	0.046	0.064	0.154	0.675	0.336	0.353
21	1.	0.634	0.366	0.655	-0.021	0.210	-0.297	-0.116	-0.063	0.058	0.110	0.127	0.599	0.343	0.094
22	1.	0.648	0.352	0.672	0.082	0.254	-0.314	-0.159	-0.033	0.089	0.018	0.171	0.642	0.311	0.090
23	1.	0.626	0.371	0.707	0.202	0.655	0.134	-0.191	-0.077	-0.015	-0.073	0.471	0.353	0.149	-0.038
24	1.	0.651	0.365	0.336	0.172	0.676	0.040	-0.223	-0.024	-0.067	-0.095	0.622	0.493	0.138	-0.037
25	1.	0.675	0.595	0.250	0.076	0.331	0.463	-0.194	0.031	0.140	0.230	0.645	0.717	0.092	0.068
26	1.	1.545	0.415	0.353	-0.085	0.237	0.495	-0.317	-0.172	0.185	0.479	0.654	-0.016	0.004	0.188
27	1.	1.614	0.346	0.150	0.097	0.507	0.413	-0.190	-0.143	-0.003	0.094	0.741	0.102	-0.005	0.125
28	1.	0.641	0.212	0.266	0.410	-0.065	0.121	0.336	0.004	0.570	-0.094	-0.038	-0.036	0.344	0.491
29	1.	0.612	0.298	0.391	0.107	0.725	0.158	0.607	0.065	0.138	-0.059	0.022	-0.064	0.579	0.306
30	1.	0.712	0.298	0.530	-0.463	0.027	0.230	0.291	-0.091	0.051	-0.556	0.049	-0.033	0.500	0.260
31	1.	0.734	0.216	0.626	-0.044	0.276	0.064	0.535	0.135	0.038	0.115	0.047	0.133	0.789	0.195
32	1.	1.792	0.201	0.613	-0.117	0.433	0.055	0.450	0.119	-0.064	0.105	0.169	0.230	0.770	0.136
33	1.	0.637	0.143	0.089	-0.093	0.150	-0.083	0.334	-0.036	-0.085	-0.011	-0.079	0.041	-0.058	0.663

0.404 0.566 0.600 0.531 0.534 0.487



## DATA SET 4

4	20	1	1	1	1.000	11.000	0.067	827.108	132.374	104.443
0.839	0.643	60.000	1.000							
1	1.	0.949	0.731		0.675	0.698	-0.164	0.932	-0.084	0.139
2	1.	0.930	0.070		0.665	-0.421	-0.558	-0.063	0.183	0.868
3	1.	0.953	0.047		0.302	-0.276	0.482	0.149	0.901	-0.035
4	1.	0.986	0.014		0.931	0.149	-0.523	0.517	0.025	0.697
5	1.	0.994	0.006		0.930	0.249	-0.260	0.667	0.563	0.020
6	1.	0.991	0.009		0.875	-0.475	-0.024	-0.001	0.693	0.499
7	1.	0.991	0.009		0.824	0.404	-0.333	0.741	-0.008	0.485
8	1.	0.963	0.037		0.802	-0.117	-2.554	0.269	0.118	0.807
9	1.	0.977	0.028		0.968	0.455	0.107	0.816	0.324	0.059
10	1.	0.991	0.009		0.907	0.064	0.390	0.517	0.724	-0.053
11	1.	0.968	0.032		0.823	-0.491	-0.222	-0.044	0.539	0.658
12	1.	0.980	0.020		0.877	-0.429	0.164	0.043	0.800	0.322
13	1.	0.948	0.057		0.800	0.061	-0.552	0.425	0.031	0.742
14	1.	0.983	0.017		0.917	0.224	0.302	0.640	0.597	-0.013
15	1.	0.990	0.010		0.878	-0.466	-0.037	0.008	0.682	0.509
16	1.	0.917	0.083		0.795	0.048	-0.532	0.411	0.049	0.728
17	1.	0.932	0.068		0.897	0.197	0.297	0.606	0.595	-0.005
18	1.	0.960	0.040		0.868	-0.452	-0.052	0.016	0.659	0.513
19	1.	0.976	0.024		0.980	-0.037	-0.119	0.432	0.472	0.466
20	1.	0.930	0.070		0.945	-0.150	-0.121	0.316	0.506	0.495

0.625 0.649 0.654

4	20	1	1	2	1	1.000	8.000	0.050	14.690	2.425	2.246
0.913	0.646	60.000	1.000								
0.888	0.434	9.000	2.000								
1	1.	0.971	0.029		0.648	0.546	-0.503	-0.017	0.944	0.024	
2	1.	0.948	0.052		0.743	-0.002	-0.184	0.069	0.009	0.902	
3	1.	0.956	0.044		0.759	0.143	0.599	0.900	0.052	0.041	
4	1.	0.977	0.023		0.871	-0.084	-0.461	-0.004	0.582	0.630	
5	1.	0.987	0.013		0.880	0.445	0.116	0.604	0.599	0.010	
6	1.	0.990	0.010		0.896	-0.306	0.106	0.622	-0.021	0.570	
7	1.	0.951	0.019		0.836	0.193	-0.493	0.010	0.779	0.393	
8	1.	0.958	0.042		0.957	-0.333	-0.135	0.048	0.331	0.766	
9	1.	0.967	0.033		0.825	0.519	-0.131	0.381	0.775	0.006	
10	1.	0.988	0.012		0.855	0.392	0.320	0.757	0.429	-0.026	
11	1.	0.964	0.034		0.965	-0.441	0.153	0.448	-0.033	0.716	
12	1.	0.974	0.024		0.975	-0.165	0.428	0.745	-0.007	0.404	
13	1.	0.966	0.034		0.855	-0.193	-0.450	-0.022	0.501	0.700	
14	1.	0.958	0.017		0.903	0.454	0.166	0.643	0.570	-0.019	
15	1.	0.994	0.006		0.949	-0.106	0.292	0.614	-0.010	0.577	
16	1.	0.938	0.067		0.849	-0.187	-0.424	-0.005	0.482	0.692	
17	1.	0.940	0.060		0.851	0.479	0.181	0.642	0.536	-0.011	
18	1.	0.957	0.043		0.903	-0.303	0.261	0.583	0.007	0.581	
19	1.	0.978	0.022		0.984	0.007	-0.022	0.447	0.428	0.465	
20	1.	0.936	0.064		0.943	-0.084	0.033	0.464	0.323	0.510	

0.651 0.629 0.658

## DATA SET 4 (cont.)

4	20	1	1	3	1	1.000	8.030	0.033	15.174	2.506	2.370
0.713	0.646	40.000	1.000								
0.188	0.613	9.000	2.000								
1	1.	0.971	0.029			0.648	0.536	-0.513	-0.016	0.944	0.023
2	1.	0.948	0.052			0.744	-0.605	-0.170	0.069	0.009	0.902
3	1.	0.956	0.044			0.754	0.156	0.597	0.900	0.051	0.042
4	1.	0.977	0.023			0.871	-0.007	-0.457	-0.004	0.582	0.629
5	1.	0.987	0.013			0.879	0.443	0.108	0.604	0.599	0.010
6	1.	0.990	0.010			0.996	-0.298	0.314	0.622	-0.021	0.570
7	1.	0.991	0.019			0.834	0.189	-0.496	0.010	0.780	0.393
8	1.	0.954	0.042			0.858	-0.339	-0.326	0.048	0.331	0.786
9	1.	0.966	0.034			0.824	0.517	-0.140	0.381	0.775	0.005
10	1.	0.989	0.012			0.854	0.400	0.314	0.757	0.429	-0.026
11	1.	0.966	0.034			0.864	-0.436	0.163	0.448	-0.033	0.716
12	1.	0.976	0.024			0.974	-0.155	0.437	0.745	-0.007	0.405
13	1.	0.967	0.033			0.854	-0.131	-0.644	-0.022	0.502	0.700
14	1.	0.988	0.012			0.867	0.459	0.158	0.643	0.570	-0.019
15	1.	0.934	0.066			0.903	-0.258	0.300	0.613	-0.010	0.577
16	1.	0.938	0.062			0.950	-0.194	-0.421	-0.005	0.483	0.692
17	1.	0.940	0.060			0.850	0.433	0.174	0.643	0.536	-0.011
18	1.	0.957	0.043			0.893	-0.294	0.269	0.587	0.007	0.581
19	1.	0.978	0.022			0.988	0.003	-0.020	0.447	0.429	0.465
20	1.	0.936	0.064			0.964	-0.081	0.036	0.464	0.324	0.510

0.651 0.628 0.658

4	20	1	7	1	1	1.000	15.000	0.050	4066.613	552.837	219.863
0.914	0.645	37.000	1.700								
0.891	0.612	22.000	2.000								
1	1.	0.961	0.039			0.849	0.302	0.384	0.937	0.000	0.009
2	1.	0.919	0.081			0.653	0.217	-0.754	0.018	0.068	0.882
3	1.	0.919	0.081			0.643	-0.671	-0.222	0.039	0.883	0.066
4	1.	0.989	0.011			0.868	0.397	-0.247	0.587	-0.020	0.635
5	1.	0.999	0.001			0.946	-0.299	0.121	0.594	0.614	0.002
6	1.	0.994	0.005			0.704	-0.319	-0.676	-0.073	0.607	0.580
7	1.	0.992	0.008			0.929	0.364	0.002	0.792	-0.000	0.390
8	1.	0.956	0.044			0.773	0.306	-0.514	0.351	0.043	0.771
9	1.	0.970	0.030			0.952	-0.067	0.245	0.769	0.391	-0.004
10	1.	0.987	0.013			0.868	-0.481	0.041	0.416	0.761	-0.027
11	1.	0.969	0.032			0.647	-0.139	-0.708	-0.022	0.432	0.719
12	1.	0.977	0.023			0.711	-0.464	-0.506	-0.010	0.732	0.415
13	1.	0.935	0.065			0.911	0.376	-0.368	0.493	-0.021	0.685
14	1.	0.954	0.045			0.922	-0.145	0.117	0.555	0.643	-0.021
15	1.	0.933	0.067			0.713	-0.104	-0.674	-0.013	0.595	0.588
16	1.	0.898	0.102			0.801	0.344	-0.368	0.475	0.002	0.669
17	1.	0.911	0.083			0.493	-0.324	0.084	0.523	0.624	0.008
18	1.	0.944	0.036			0.707	-0.287	-0.614	-0.005	0.576	0.587
19	1.	0.975	0.014			0.935	-0.077	-0.309	0.423	0.439	0.467
20	1.	0.924	0.074			0.864	-0.113	-0.409	0.300	0.455	0.521

0.626 0.650 0.654

4	20	3	2	2	1	1.000	3.000	0.033	14.722	2.457	2.778
7.910	0.546	60.000	1.000								
7.946	0.633	12.000	2.000								
1	1.	0.979	0.021	0.649	0.543	-0.513			-0.016	0.949	0.022
2	1.	0.962	0.038	0.745	-0.613	-0.177			0.068	0.007	0.912
3	1.	0.964	0.032	0.761	0.153	0.605			0.909	0.048	0.040
4	1.	0.976	0.021	0.871	-0.093	-0.460			-0.002	0.585	0.630
5	1.	0.986	0.014	0.880	0.447	0.110			0.605	0.599	0.012
6	1.	0.990	0.011	0.895	-0.301	0.310			0.623	-0.020	0.571
7	1.	0.987	0.018	0.836	0.193	-0.436			0.012	0.782	0.393
8	1.	0.973	0.027	0.857	-0.340	-0.333			0.048	0.333	0.791
9	1.	0.971	0.027	0.824	0.521	-0.139			0.383	0.779	0.004
10	1.	0.987	0.013	0.855	0.396	0.315			0.758	0.429	-0.023
11	1.	0.971	0.029	0.866	-0.441	0.161			0.452	-0.034	0.720
12	1.	0.973	0.021	0.876	-0.157	0.432			0.749	-0.007	0.406
13	1.	0.971	0.029	0.856	-0.190	-0.450			-0.022	0.505	0.702
14	1.	0.987	0.013	0.868	0.456	0.160			0.644	-0.570	-0.017
15	1.	0.991	0.009	0.902	-0.390	0.296			0.615	-0.007	0.578
16	1.	0.997	0.000	0.852	-0.195	-0.432			-0.009	0.489	0.669
17	1.	0.953	0.047	0.853	0.439	0.179			0.650	0.541	-0.016
18	1.	0.966	0.036	0.894	-0.302	0.270			0.588	0.004	0.584
19	1.	0.978	0.027	0.989	0.008	-0.021			0.450	0.430	0.466
20	1.	0.943	0.057	0.967	-0.084	0.035			0.469	0.324	0.513

0.651 0.829 0.657

4	20	3	2	2	1	1.000	3.000	0.033	15.137	2.524	2.339
7.910	0.644	60.000	1.000								
7.946	0.633	12.000	2.000								
1	1.	0.979	0.021	0.649	0.517	-0.512			-0.016	0.949	0.022
2	1.	0.962	0.038	0.746	-0.614	-0.169			0.068	0.007	0.912
3	1.	0.968	0.032	0.760	0.161	0.404			0.909	0.048	0.041
4	1.	0.974	0.027	0.872	-0.094	-0.454			-0.001	0.585	0.630
5	1.	0.984	0.014	0.879	0.442	0.105			0.605	0.599	0.012
6	1.	0.988	0.012	0.895	-0.295	0.315			0.623	-0.020	0.571
7	1.	0.982	0.014	0.836	0.184	-0.497			0.017	0.782	0.393
8	1.	0.965	0.035	0.860	-0.143	-0.328			0.047	0.334	0.791
9	1.	0.973	0.027	0.875	0.521	-0.144			0.383	0.779	0.004
10	1.	0.987	0.013	0.854	0.401	0.311			0.758	0.429	-0.023
11	1.	0.971	0.029	0.867	-0.434	0.167			0.451	-0.034	0.720
12	1.	0.977	0.021	0.875	-0.153	0.435			0.748	-0.007	0.407
13	1.	0.971	0.029	0.857	-0.194	-0.467			-0.022	0.505	0.702
14	1.	0.987	0.013	0.867	0.459	0.155			0.644	0.570	-0.017
15	1.	0.991	0.009	0.902	-0.294	0.301			0.615	-0.007	0.578
16	1.	0.941	0.049	0.853	-0.199	-0.429			-0.009	0.489	0.699
17	1.	0.953	0.047	0.853	0.443	0.175			0.650	0.540	-0.016
18	1.	0.966	0.036	0.894	-0.298	0.274			0.588	0.004	0.585
19	1.	0.978	0.027	0.989	0.007	-0.020			0.450	0.430	0.466
20	1.	0.943	0.057	0.967	-0.082	0.038			0.468	0.325	0.513

0.651 0.829 0.657



DATA SET 5

1	2	1	1	1	0.999	71.000	0.050	27.433	9.006
1.111	0.984	13.000	1.000						
1	1.	0.830	0.170	0.880	0.236			0.780	0.095
2	1.	0.899	0.107	0.974	0.359			0.865	-0.022
3	1.	0.874	0.126	0.946	0.343			0.934	-0.017
4	1.	0.801	0.199	0.854	0.267			0.782	0.062
5	1.	0.910	0.090	0.704	-0.644			0.023	0.854
6	1.	0.837	0.363	0.589	-0.539			0.019	0.714
7	1.	0.984	0.416	0.326	-0.555			-0.035	0.706
8	1.	0.443	0.937	0.574	-0.366			0.134	0.547
		0.444	0.584						

1	2	1	2	1	0.999	14.000	0.033	4.440	1.510
1.115	0.989	14.000	1.000						
1	1.	0.839	0.167	0.856	0.324			0.791	0.072
2	1.	0.889	0.111	0.949	0.411			0.854	-0.010
3	1.	0.820	0.180	0.809	0.409			0.827	-0.025
4	1.	0.808	0.192	0.831	0.342			0.789	0.045
5	1.	0.887	0.111	0.750	-0.571			0.029	0.837
6	1.	0.840	0.360	0.631	-0.492			0.015	0.714
7	1.	0.983	0.417	0.549	-0.510			-0.038	0.704
8	1.	0.492	0.509	0.607	-0.351			0.110	0.577
		0.989	0.989						

1	2	1	1	1	0.999	9.000	0.050	5.938	2.067
1.117	0.990	24.000	1.000						
1	1.	0.838	0.162	0.913	-0.470			0.793	0.065
2	1.	0.891	0.109	0.903	-0.496			0.951	-0.005
3	1.	0.819	0.181	0.748	-0.404			0.826	-0.027
4	1.	0.807	0.193	0.787	-0.432			0.789	0.047
5	1.	0.890	0.170	0.405	0.482			0.030	0.831
6	1.	0.816	0.341	0.680	0.477			0.014	0.713
7	1.	0.997	0.418	0.621	0.444			-0.038	0.702
8	1.	0.510	0.500	0.645	0.290			0.105	0.583
		0.990	0.190						

## DATA SET 5 (cont.)

S	A	2	2	1	1	1-000	11-000	0-050	2746.550	1256.756
1.115	0.970	18.000	1.000							
1	1.	0.765	0.255			0.778	0.374		0.127	0.714
2	1.	0.999	0.001			0.800	0.600		-0.047	0.921
3	1.	0.780	0.220			0.737	0.457		0.011	0.791
4	1.	0.702	0.298			0.743	0.384		0.096	0.709
5	1.	0.929	0.001			0.856	-0.516		0.908	-0.015
6	1.	0.583	0.417			0.675	-0.357		0.674	0.031
7	1.	0.533	0.467			0.677	-0.375		0.662	-0.009
8	1.	0.433	0.567			0.634	-0.160		0.491	0.183
		0.969	0.971							
S	A	2	2	2	1	0.979	4.000	0.033	4.673	1.771
1.090	0.789	14.000	1.000							
1	1.	0.877	0.123			0.859	0.373		0.834	0.063
2	1.	0.903	0.097			0.841	0.442		0.876	-0.001
3	1.	0.872	0.128			0.813	0.459		0.871	-0.030
4	1.	0.861	0.139			0.829	0.396		0.839	0.038
5	1.	0.850	0.150			0.758	-0.574		0.078	0.816
6	1.	0.739	0.261			0.675	-0.533		0.018	0.785
7	1.	0.717	0.283			0.614	-0.580		-0.055	0.809
8	1.	0.625	0.375			0.671	-0.418		0.104	0.681
		0.589	0.989							
S	A	2	2	3	1	0.574	6.000	0.033	5.770	2.230
1.090	0.989	24.000	1.000							
1	1.	0.877	0.123			0.821	0.450		0.836	0.063
2	1.	0.903	0.099			0.806	0.510		0.873	0.002
3	1.	0.871	0.129			0.769	0.529		0.871	-0.030
4	1.	0.862	0.138			0.800	0.471		0.841	0.035
5	1.	0.830	0.170			0.795	-0.544		0.084	0.803
6	1.	0.731	0.269			0.716	-0.467		0.021	0.779
7	1.	0.714	0.284			0.664	-0.524		-0.057	0.800
8	1.	0.653	0.347			0.717	-0.373		0.099	0.700
		0.589	0.989							

## DATA SET 6

6	12	3	1	1	1	0.995100.000	0.100	22.715	4.413	2.006
0.974	0.791	29.000	1.000							
1	1.	0.935	0.065			0.942	-0.217	0.002		
2	1.	0.626	0.374			0.703	-0.226	0.283		
3	1.	0.343	0.657			0.352	0.364	0.295		
4	1.	0.489	0.511			0.399	0.509	0.266		
5	1.	0.760	0.240			0.869	0.070	-0.022		
6	1.	0.632	0.368			0.354	0.557	0.444		
7	1.	0.277	0.723			0.145	0.396	0.315		
8	1.	0.159	0.841			0.286	0.153	0.231		
9	1.	0.028	0.972			0.092	0.080	0.117		
10	1.	0.016	0.984			0.110	-0.041	0.051		
11	1.	0.694	0.306			0.796	0.120	-0.157		
12	1.	0.793	0.207			0.563	0.578	-0.377		
		0.771	0.725	0.876						

6	12	3	1	2	1	0.98115.000	0.033	3.893	1.204	0.924
0.979	0.377	12.000	1.000							
1	1.	0.809	0.101			0.812	-0.466	-0.146		
2	1.	0.425	0.575			0.598	-0.255	-0.045		
3	1.	0.316	0.684			0.485	0.284	-0.001		
4	1.	0.568	0.432			0.577	0.476	-0.097		
5	1.	0.766	0.234			0.842	-0.227	-0.081		
6	1.	0.562	0.438			0.562	0.495	0.034		
7	1.	0.315	0.685			0.301	0.471	-0.056		
8	1.	0.365	0.635			0.375	0.114	0.460		
9	1.	0.363	0.637			0.187	-0.009	0.572		
10	1.	0.366	0.634			0.153	-0.147	0.567		
11	1.	0.702	0.298			0.806	-0.230	-0.025		
12	1.	0.374	0.626			0.588	0.097	-0.132		
		0.890	0.852	0.990						





## DATA SET 7

7	15	5	1	1	1	0.993	92.000	0.167	10.868	4.258	2.938	2.298	1.301
0.479	0.556	60.000	1.000										
1	1.	0.815	0.195	0.819	0.152	0.041	0.325	0.116			0.714	0.067	0.092
2	1.	0.705	0.295	0.557	0.414	-0.156	-0.440	0.068			0.102	0.024	0.112
3	-1.	0.385	0.615	-0.274	-0.241	-0.325	-0.070	0.376			-0.035	-0.054	0.040
4	1.	0.421	0.579	0.418	0.031	-0.372	-0.323	0.044			0.046	-0.002	0.370
5	1.	0.331	0.619	0.477	-0.272	-0.269	0.033	-0.077			0.195	0.077	0.507
6	1.	0.676	0.324	0.751	0.029	-0.162	0.284	0.059			0.622	-0.013	0.259
7	1.	0.458	0.542	0.543	0.190	-0.274	-0.222	0.030			0.170	-0.047	0.283
8	-1.	0.570	0.430	-0.274	-0.530	-0.234	0.012	0.394			-0.038	0.133	-0.163
9	1.	0.140	0.860	0.123	-0.259	-0.223	-0.060	0.041			0.005	0.058	0.312
10	1.	0.779	0.261	0.437	-0.623	-0.307	-0.081	-0.241			-0.049	0.241	0.834
11	1.	0.314	0.636	0.546	0.092	-0.061	0.055	0.007			0.339	0.044	0.174
12	1.	0.279	0.721	0.275	0.283	0.199	-0.333	0.012			-0.045	0.133	-0.030
13	-1.	0.272	0.728	-0.050	-0.322	0.129	0.005	0.386			0.066	0.320	-0.016
14	1.	0.341	0.657	0.255	0.025	0.205	-0.361	0.321			-0.003	0.404	-0.022
15	1.	0.793	0.207	0.567	-0.403	0.526	-0.181	-0.003			0.007	0.792	0.346

0.775 0.548 0.651 0.666 0.639

7	15	5	1	2	1	0.981	35.000	0.087	3.398	1.537	0.911	0.778	0.613
1.776	0.580	45.000	1.000										
1	1.	0.645	0.355	0.739	0.032	-0.300	-0.069	-0.057			0.677	-0.071	0.054
2	1.	0.733	0.267	0.669	0.254	0.394	0.114	-0.170			0.241	0.074	0.704
3	-1.	0.447	0.553	-0.274	-0.410	0.303	0.138	-0.305			-0.086	0.574	0.187
4	1.	0.437	0.563	0.453	-0.161	0.341	0.227	-0.026			0.290	0.268	0.480
5	1.	0.519	0.481	0.464	-0.392	-0.157	0.263	-0.237			0.661	0.374	0.075
6	1.	0.598	0.402	0.709	-0.123	-0.279	0.030	-0.073			0.708	0.019	0.032
7	1.	0.486	0.514	0.631	0.012	0.202	0.167	0.137			0.330	0.001	0.390
8	-1.	0.493	0.507	-0.398	-0.593	0.147	-0.094	-0.100			-0.046	0.513	-0.064
9	1.	0.625	0.375	0.171	-0.459	0.212	-0.051	0.580			0.035	0.027	-0.005
10	1.	0.456	0.544	0.384	-0.540	-0.111	0.051	0.006			0.549	0.334	-0.025
11	1.	0.360	0.640	0.555	0.036	-0.074	-0.101	0.121			0.381	-0.109	0.118
12	1.	0.296	0.704	0.325	0.327	0.233	-0.134	-0.104			0.011	-0.053	0.402
13	-1.	0.255	0.745	-0.123	-0.309	0.032	-0.347	-0.154			0.014	0.314	-0.041
14	1.	0.451	0.549	0.297	0.024	0.382	-0.471	-0.076			-0.035	0.147	0.424
15	1.	0.457	0.543	0.420	-0.147	-0.164	-0.455	-0.123			0.430	0.131	0.017

0.615 0.590 0.553 0.514 0.444







## DATA SET 8

	8	17	5	1	1	1	0.998	71.000	0.150	22.745	7.152	5.225	2.405	1.819
1.212	0.875	13.000	1.100											
1	1.	0.201	0.799	0.432	-0.050	0.072	-0.064	0.046	0.131	-0.019	0.095	0.141	0.108	
2	1.	0.111	0.389	0.288	0.024	0.012	-0.148	0.076	0.050	0.039	-0.022	0.195	0.089	
3	1.	0.839	0.170	0.819	-0.278	-0.266	0.399	-0.023	0.626	-0.048	0.007	-0.009	0.117	
4	1.	0.864	0.176	0.847	-0.267	-0.228	-0.015	-0.154	0.680	0.004	0.055	0.100	-0.027	
5	1.	0.772	0.228	0.805	-0.011	-0.331	0.032	-0.113	0.617	0.238	0.001	0.035	0.022	
6	1.	0.553	0.407	0.598	-0.013	0.137	-0.457	0.057	0.074	0.019	0.050	0.563	0.057	
7	1.	0.697	0.313	0.556	0.030	0.260	-0.608	0.051	-0.033	0.033	0.029	0.706	0.004	
8	1.	0.434	0.566	0.528	-0.132	0.085	-0.361	-0.038	0.194	-0.046	0.019	0.443	-0.034	
9	1.	0.627	0.373	0.349	0.706	-0.056	0.029	-0.023	-0.024	0.694	0.132	0.905	0.091	
10	1.	0.769	0.231	0.222	0.815	-0.194	-0.067	-0.090	-0.009	0.841	0.016	0.058	-0.031	
11	1.	0.682	0.318	0.268	0.736	-0.254	0.030	-0.061	0.059	0.778	-0.002	-0.035	0.019	
12	1.	0.550	0.450	0.581	0.152	0.436	0.081	-0.124	0.033	0.114	0.555	0.049	0.043	
13	1.	0.829	0.171	0.565	0.042	0.959	0.180	-0.204	0.004	-0.029	0.787	-0.023	0.002	
14	1.	0.503	0.497	0.606	0.220	0.273	0.113	0.909	0.046	0.182	0.404	0.014	0.176	
15	1.	0.733	0.267	0.724	0.077	0.127	0.138	0.410	-0.014	-0.020	0.127	0.023	0.566	
16	1.	0.642	0.358	0.683	0.048	-0.050	0.109	0.399	0.081	0.038	-0.023	0.021	0.527	
17	1.	0.534	0.466	0.634	0.119	0.173	0.111	0.273	-0.013	0.039	0.100	0.030	0.419	

0.868 0.906 0.362 0.882 0.855

	8	17	5	1	2	1	0.994	16.000	0.067	5.910	1.949	1.083	0.941	0.664
1.380	0.343	25.000	1.000											
1	1.	0.440	0.560	0.463	-0.131	0.011	-0.259	0.374	0.110	-0.030	0.154	0.005	0.513	
2	1.	0.570	0.430	0.337	-0.063	-0.023	-0.401	0.460	-0.004	0.051	-0.011	0.038	0.694	
3	1.	0.843	0.157	0.741	-0.287	-0.431	0.148	-0.071	0.807	-0.066	0.054	0.020	-0.034	
4	1.	0.755	0.205	0.763	-0.286	-0.351	0.010	-0.036	0.721	-0.043	0.014	0.142	0.031	
5	1.	0.752	0.248	0.757	-0.015	-0.410	0.027	-0.102	0.705	0.217	-0.003	0.097	-0.002	
6	1.	0.624	0.376	0.624	-0.149	0.213	-0.264	0.312	0.137	0.057	0.025	0.586	-0.030	
7	1.	0.634	0.366	0.563	-0.124	0.322	-0.372	-0.278	-0.025	0.064	0.009	0.643	0.066	
8	1.	0.443	0.557	0.537	-0.234	0.074	-0.230	0.202	0.220	-0.043	-0.020	0.441	0.035	
9	1.	0.633	0.367	0.421	0.671	0.020	0.030	-0.066	0.033	0.723	0.182	0.077	-0.064	
10	1.	0.773	0.227	0.303	0.806	-0.059	-0.148	-0.070	-0.038	0.863	-0.007	0.071	0.033	
11	1.	0.683	0.317	0.325	0.736	-0.153	-0.107	0.023	0.062	0.800	0.010	-0.033	0.093	
12	1.	0.522	0.472	0.556	0.037	0.362	0.176	0.057	0.018	0.071	0.540	0.077	0.020	
13	1.	0.630	0.371	0.581	-0.097	0.244	0.120	0.036	-0.034	-0.085	0.557	0.739	0.036	
14	1.	0.569	0.451	0.646	0.130	0.225	0.252	0.032	0.156	0.182	0.500	0.027	-0.064	
15	1.	0.556	0.406	0.738	-0.013	0.019	0.169	0.156	0.352	0.089	0.425	-0.049	0.133	
16	1.	0.506	0.406	0.681	-0.015	-0.129	0.111	0.105	0.437	0.117	0.270	-0.040	0.111	
17	1.	0.474	0.525	0.559	0.024	0.077	0.167	0.110	0.264	0.110	0.412	-0.013	0.084	

1.614 1.020 0.420 0.882 0.364



## DATA SET 3 (cont.)

	4	17	5	2	2	1	0.044	4.000	0.033	6.314	2.253	1.406	1.268	1.109
1.007	0.546	17.000	1.000											
1	1.	0.699	0.301				0.493	-0.181	0.098	-0.417	0.489	0.129	-0.036	0.173
2	1.	0.793	0.207				0.352	-0.089	0.170	-0.647	0.462	-0.008	0.082	-0.329
3	1.	0.851	0.149				0.733	-0.274	0.414	0.247	-0.080	0.836	-0.079	0.044
4	1.	0.870	0.130				0.762	-0.286	0.366	0.103	-0.114	0.760	-0.054	0.015
5	1.	0.803	0.193				0.760	-0.006	0.433	0.120	-0.146	0.758	0.216	-0.073
6	1.	0.728	0.272				0.665	-0.177	-0.137	-0.270	-0.411	0.143	0.073	0.081
7	1.	0.752	0.248				0.565	-0.154	-0.307	-0.421	-0.370	-0.049	0.088	0.086
8	1.	0.633	0.367				0.572	-0.298	-0.044	-0.276	-0.372	0.234	-0.040	-0.045
9	1.	0.750	0.250				0.433	0.745	-0.032	-0.009	-0.082	0.042	0.797	0.194
10	1.	0.829	0.171				0.305	0.630	0.070	-0.178	-0.102	-0.040	0.901	0.003
11	1.	0.733	0.267				0.337	0.795	0.112	-0.135	-0.077	0.072	0.867	-0.012
12	1.	0.665	0.335				0.627	0.344	-0.433	0.116	0.133	-0.002	0.066	0.691
13	1.	0.759	0.241				0.601	-0.097	-0.565	0.157	0.212	-0.030	-0.102	0.778
14	1.	0.643	0.357				0.576	0.155	-0.308	0.251	0.069	0.193	0.187	0.612
15	1.	0.662	0.338				0.766	-0.016	0.007	0.192	0.196	0.459	0.081	0.450
16	1.	0.607	0.393				0.716	-0.011	0.204	0.190	0.124	0.579	0.112	0.271
17	1.	0.504	0.438				0.695	0.033	-0.083	0.195	0.176	0.354	0.102	0.450

3.892 0.478 0.993 0.884 0.964

	4	17	5	2	2	1	0.556	12.000	3.033	3.903	2.897	1.890	1.781	1.525
0.556	0.817	19.000	1.000											
1	1.	0.719	0.281				0.401	-0.123	0.305	0.117	0.569	0.129	-0.035	0.159
2	1.	0.757	0.231				0.345	-0.107	0.462	0.304	0.574	-0.006	0.086	-0.032
3	1.	0.821	0.179				0.714	-0.254	0.230	-0.349	-0.202	0.816	-0.092	0.061
4	1.	0.791	0.209				0.745	-0.281	0.270	-0.209	-0.195	0.731	-0.055	0.003
5	1.	0.774	0.224				0.744	-0.001	0.327	-0.230	-0.249	0.732	0.216	-0.034
6	1.	0.712	0.283				0.645	-0.198	-0.329	0.416	-0.246	0.141	0.076	0.133
7	1.	0.723	0.277				0.567	-0.141	-0.053	0.576	-0.181	-0.057	0.093	0.119
8	1.	0.662	0.338				0.573	-0.332	0.104	0.358	-0.280	0.223	-0.037	-0.044
9	1.	0.756	0.244				0.428	0.744	0.021	0.096	-0.044	0.050	0.401	0.118
10	1.	0.820	0.190				0.205	0.811	0.193	0.194	-0.067	-0.038	0.896	0.107
11	1.	0.749	0.211				0.321	0.793	0.255	0.085	-0.078	0.072	0.864	-0.014
12	1.	0.676	0.324				0.634	0.059	-0.477	0.123	0.165	-0.019	0.065	0.702
13	1.	0.749	0.252				0.600	-0.073	-0.554	0.043	0.234	-0.021	-0.103	0.774
14	1.	0.651	0.342				0.686	0.141	-0.378	-0.365	0.020	0.206	0.188	0.616
15	1.	0.683	0.314				0.775	0.007	-0.057	-0.261	0.122	0.506	0.071	0.442
16	1.	0.567	0.355				0.723	0.002	0.122	-0.123	0.073	0.633	0.103	0.244
17	1.	0.503	0.401				0.714	0.052	-0.147	-0.216	0.133	0.404	0.089	0.494

1.714 0.981 0.719 0.916 1.417

## DATA SET 9

0	17	5	1	1	1	0.598	59.000	0.117	20.908	0.244	4.377	2.745	1.754
1.186	0.961	26.000	1.000										
1	1.	0.390	0.700	0.437	-0.034	0.017	0.210	0.252					
2	1.	0.195	0.315	0.403	0.097	3.041	0.042	0.367					
3	1.	0.819	0.191	0.924	-0.291	-0.081	-0.214	-0.052					
4	1.	0.863	0.140	0.948	-0.279	-0.055	-0.149	-0.112					
5	1.	0.742	0.159	0.790	-0.205	-0.140	-0.192	-0.070					
6	1.	1.722	1.272	0.666	-0.049	-0.054	0.526	-0.170					
7	1.	1.642	1.153	0.627	0.124	-1.159	0.666	-0.174					
8	1.	0.500	0.533	0.470	-0.132	-0.060	0.353	-0.140					
9	1.	0.589	0.417	0.313	0.626	-0.294	-0.051	-0.093					
10	1.	1.654	0.244	0.244	0.644	-0.404	-0.126	-0.011					
11	1.	0.718	0.282	0.439	0.679	-0.156	-0.105	-0.059					
12	1.	0.804	0.196	0.494	0.365	0.636	-0.021	-0.115					
13	1.	0.445	0.415	0.534	0.241	0.269	0.036	-0.017					
14	1.	0.531	0.449	0.579	0.362	0.225	-0.006	-0.148					
15	1.	0.531	0.449	0.537	0.150	0.055	0.062	0.333					
16	1.	0.711	0.290	0.673	-0.046	-0.043	-0.013	0.510					
17	1.	1.454	0.144	0.564	0.186	-1.030	0.195	0.239					

0.017 0.018 0.019 0.020

0	17	5	1	2	1	0.039	31.000	0.067	5.064	1.660	1.097	0.794	0.589
1.184	0.956	27.300	1.000										
1	1.	0.339	0.662	0.476	-0.121	0.026	0.169	0.259					
2	1.	0.204	0.796	0.476	-0.013	0.075	0.069	0.197					
3	1.	0.815	0.145	0.763	-0.124	-0.144	-0.341	-0.194					
4	1.	0.813	0.147	0.773	-0.127	-0.137	-0.289	-0.145					
5	1.	0.755	0.245	0.734	-0.229	-0.204	-0.319	-0.140					
6	1.	0.723	0.277	0.649	-0.175	-0.137	0.435	-0.138					
7	1.	0.635	0.375	0.647	-0.069	-0.203	0.373	-0.091					
8	1.	0.509	0.491	0.582	-0.227	-0.133	0.281	-0.144					
9	1.	0.593	0.417	0.387	0.624	-0.173	-0.002	-0.063					
10	1.	0.653	0.350	0.305	0.690	-1.244	-0.105	-0.064					
11	1.	0.729	0.290	0.407	0.705	-0.279	-0.054	-0.041					
12	1.	0.734	0.266	0.519	0.185	0.662	-0.064	-0.220					
13	1.	0.537	0.473	0.575	0.116	0.593	0.030	-0.112					
14	1.	0.624	0.374	0.524	0.272	0.243	-0.021	0.047					
15	1.	0.713	0.287	0.674	0.044	0.031	0.006	0.273					
16	1.	0.706	0.294	0.672	-0.164	-1.014	-0.125	0.469					
17	1.	0.454	0.556	0.634	0.064	0.036	0.120	0.131					

0.019 0.019 0.140 0.440 0.870





## DATA SET 10

10	3	1	1	1	0.000	8.000	0.017	10820.321	3581.879	3583.879	
0.826	0.676	50.000	1.000								
0.821	0.670	9.000	2.000								
0.819	0.669	4.000	3.000								
0.818	0.668	5.000	4.000								
0.818	0.668	5.000	5.000								
0.818	0.668	5.000	6.000								
0.817	0.667	5.000	7.000								
0.817	0.667	5.000	8.000								
0.817	0.667	5.000	9.000								
0.817	0.667	4.000	10.000								
0.817	0.667	5.000	11.000								
0.817	0.667	4.000	12.000								
0.817	0.667	4.000	13.000								
0.817	0.667	4.000	14.000								
0.817	0.667	4.000	15.000								
0.817	0.667	4.000	16.000								
0.817	0.667	3.000	17.000								
0.817	0.667	2.000	18.000								
0.817	0.667	3.000	19.000								
0.817	0.667	2.000	20.000								
0.817	0.667	2.000	20.000								
1	1.	0.000	0.001		0.754	0.450	0.000				-0.001 0.300 0.923
2	1.	0.000	0.001		0.816	0.511	-0.260				-0.001 0.706 0.706
3	1.	0.000	0.001		0.754	0.254	-0.547				-0.300 0.923 0.392
4	1.	0.000	0.001		0.754	-0.247	-0.404				0.442 0.923 -0.001
5	1.	0.000	0.001		0.815	-0.444	-0.300				0.706 0.706 -0.001
6	1.	0.000	0.001		0.754	-0.655	0.334				0.923 0.182 -0.001
7	1.	0.000	0.001		0.754	-0.404	0.514				0.923 -0.001 0.392
8	1.	0.000	0.001		0.816	-0.022	0.577				0.706 -0.001 0.706
9	1.	0.000	0.001		0.754	0.361	0.544				0.382 -0.001 0.923
0.867	0.667	0.667	0.667								





## DATA SET 10 (cont.)

10	9	3	1	3	1	0.493	10.000	0.033	5.417	1.793	1.793
0.828	0.676	60.000	1.000								
0.921	0.670	9.000	2.000								
0.819	0.669	4.000	3.000								
0.818	0.668	5.000	4.000								
0.818	0.668	5.000	5.000								
0.818	0.668	5.000	6.000								
0.817	0.667	5.000	7.000								
0.817	0.667	5.000	8.000								
0.817	0.667	5.000	9.000								
0.817	0.667	5.000	10.000								
0.817	0.667	5.000	11.000								
0.817	0.667	4.000	12.000								
0.817	0.667	4.000	13.000								
0.817	0.667	4.000	14.000								
0.817	0.667	4.000	15.000								
0.817	0.667	4.000	16.000								
0.817	0.667	3.000	17.000								
0.817	0.667	2.000	18.000								
0.817	0.667	3.000	19.000								
0.817	0.667	2.000	20.000								
1	1.	0.699	0.001			0.754	0.650	-0.390	-0.001	0.382	0.923
2	1.	0.699	0.001			0.814	0.511	0.260	-0.001	0.706	0.706
3	1.	0.699	0.001			0.754	0.294	0.537	-0.001	0.923	0.382
4	1.	0.699	0.001			0.754	-0.247	0.608	0.382	0.923	-0.001
5	1.	0.699	0.001			0.816	-0.488	0.308	0.706	0.706	-0.001
6	1.	0.699	0.001			0.754	-0.455	-0.033	0.923	0.382	-0.001
7	1.	0.699	0.001			0.754	-0.403	-0.519	0.923	-0.001	0.382
8	1.	0.699	0.001			0.816	-0.023	-0.577	0.706	-0.001	0.706
9	1.	0.699	0.001			0.754	0.361	-0.543	0.382	-0.001	0.923
0.667	0.667	0.667	0.667								

[illegible]

## DATA SET 10 (cont.)

10	9	3	2	2	1	0.000	3.000	0.050	5.415	1.792	1.792
0.828	0.675	60.000	1.000								
0.821	0.670	9.000	2.000								
0.819	0.669	4.000	3.000								
0.816	0.668	5.000	4.000								
0.813	0.668	5.000	5.000								
0.818	0.669	5.000	6.000								
0.817	0.667	5.000	7.000								
0.817	0.667	5.000	8.000								
0.817	0.667	5.000	9.000								
0.817	0.667	4.000	10.000								
0.817	0.667	5.000	11.000								
0.817	0.667	4.000	12.000								
0.817	0.667	4.000	13.000								
0.817	0.667	4.000	14.000								
0.817	0.667	4.000	15.000								
0.817	0.667	3.000	16.000								
0.817	0.667	2.000	17.000								
0.817	0.667	3.000	18.000								
0.817	0.667	2.000	19.000								
0.817	0.667	3.000	20.000								
1	1.	0.999	0.001			0.754	-0.650	0.090		-0.001	0.923
2	1.	0.999	0.001			0.816	-0.511	-0.260		-0.001	0.706
3	1.	0.999	0.001			0.754	-0.293	-0.587		-0.001	0.923
4	1.	0.999	0.001			0.754	0.247	-0.608		0.382	0.923
5	1.	0.999	0.001			0.816	0.480	-0.303		0.706	0.706
6	1.	0.999	0.001			0.754	0.655	0.019		0.923	0.392
7	1.	0.999	0.001			0.754	0.403	0.514		0.923	-0.001
8	1.	0.999	0.001			0.816	0.027	0.577		0.706	-0.001
9	1.	0.999	0.001			0.754	-0.362	0.547		0.382	-0.001
0.667	0.667	0.667	0.667								



## DATA SET 11

11	9	1	1	1	1	1	0.009	13.000	0.017	14.720
1	1.	0.558	0.342				0.811			
2	1.	0.641	0.359				0.812			
3	1.	0.225	0.772				0.477			
4	1.	0.168	0.832				0.410			
5	1.	0.454	0.546				0.674			
6	1.	0.800	0.200				0.394			
7	1.	0.795	0.205				0.840			
8	1.	0.424	0.576				0.659			
9	1.	0.703	0.297				0.838			
11	9	1	1	2	1	1	0.008	9.000	0.033	4.810
1	1.	0.653	0.347				0.508			
2	1.	0.653	0.347				0.812			
3	1.	0.224	0.776				0.473			
4	1.	0.169	0.831				0.411			
5	1.	0.452	0.548				0.672			
6	1.	0.801	0.199				0.395			
7	1.	0.710	0.290				0.843			
8	1.	0.424	0.576				0.659			
9	1.	0.707	0.293				0.841			
11	9	1	1	3	1	1	0.007	7.000	0.033	3.000
1	1.	0.632	0.362				0.799			
2	1.	0.655	0.345				0.810			
3	1.	0.215	0.785				0.463			
4	1.	0.165	0.835				0.407			
5	1.	0.467	0.533				0.694			
6	1.	0.803	0.197				0.394			
7	1.	0.724	0.276				0.851			
8	1.	0.424	0.576				0.659			
9	1.	0.721	0.279				0.849			



12	5	1	1	1	1	0.581100.000	0.017	801.268
1	1.	0.769	0.001			0.999		
2	1.	0.803	0.137			0.945		
3	1.	0.705	0.295			0.440		
4	1.	0.539	0.461			0.734		
5	1.	0.396	0.524			0.630		
12	5	1	1	2	1	0.985 11.000	0.017	3.351
1	1.	0.509	0.001			1.000		
2	1.	0.822	0.178			0.907		
3	1.	0.643	0.352			0.905		
4	1.	0.465	0.505			0.704		
5	1.	0.343	0.637			0.603		
12	5	1	1	3	1	0.587 8.000	0.017	5.031
1	1.	0.509	0.001			1.000		
2	1.	0.914	0.186			0.502		
3	1.	0.643	0.357			0.802		
4	1.	0.463	0.507			0.702		
5	1.	0.342	0.638			0.602		
12	5	1	2	1	1	1.000 5.000	0.017	2012.124
1	1.	0.509	0.001			1.000		
2	1.	0.854	0.106			0.945		
3	1.	0.705	0.224			0.940		
4	1.	0.540	0.447			0.735		
5	1.	0.357	0.603			0.630		
12	5	1	2	2	1	0.583 4.000	0.017	3.544
1	1.	0.543	0.022			0.994		
2	1.	0.437	0.163			0.915		
3	1.	0.722	0.249			0.455		
4	1.	0.615	0.385			0.744		
5	1.	0.402	0.509			0.701		
12	5	1	2	3	1	0.535 6.000	0.033	5.000
1	1.	0.547	0.114			0.972		
2	1.	0.700	0.231			0.334		
3	1.	0.713	0.291			0.467		
4	1.	0.522	0.474			0.791		
5	1.	0.343	0.642			0.701		

## CHAPTER 13

### DISCUSSION AND CONCLUSIONS

#### 13.1 The Six Special Cases

Before discussing the results of the analyses of the twelve data sets it may be useful to relate the special cases of the loss and scaling function parameters to traditional factor analysis techniques. For convenience, we shall construct six five-letter acronyms. The first two letters will indicate the scaling parameter and the last three the loss parameter.

If the scaling parameter is  $p = 0$ , the scaling matrix involves only the Residual variance and the first two letters of the acronym for this case will be RE. If the scaling parameter is  $p = .5$ , the scaling matrix involves the sum of the residual and estimated or common variances which is the Total variance, and the first two letters of the acronym for this case will be TO. If the scaling parameter is  $p = 1$ , the scaling matrix involves only the Estimated or common variances and the first two letters of the acronym for this case will be ES.

If the loss parameter is  $P_W = 1$ , the loss function involves only the residual COVariances and the last three letters of the acronym for this case will be COV. If the loss parameter is  $P_W = 0$ , then the loss function involves both residual Variances and Covariances, equally weighted, and the last three letters of the acronym for this case will be VAC. Thus we shall have the following cases:

$p = 0,$	$P_W = 1.$	RECOV
$p = .5,$	$P_W = 1.$	TOCOV
$p = 1.,$	$P_W = 1.$	ESCOV
$p = 0,$	$P_W = 0$	REVAC
$p = .5,$	$P_W = 0$	TOVAC
$p = 1.,$	$P_W = 0$	ESVAC

RECOV factor analysis is closely associated with maximum likelihood factor analysis developed by Lawley (1940) and with canonical factor analysis developed



by Rao (1955). As a matter of fact, the equations to be satisfied by RECOV and the latter two are equivalent. TOCOV factor analysis is closely associated with minres factor analysis developed by Harmon (1967). ESCOV factor analysis is similar to alpha factor analysis developed by Kaiser and Caffrey (1965). REVAC factor analysis has been discussed by Anderson and Rubin (1956) who have pointed out fundamental difficulties with the model. These we have met by the imposition of somewhat unsophisticated computational constraints. TOVAC factor analysis is the same as what many investigators call principal components analysis. Actually, any of the methods of factor analysis which is a special case of the loss and scaling parameters we have discussed may be regarded as a principal components analysis of a real symmetric matrix, whether or not all its roots are non-negative. ESVAC factor analysis, to our knowledge, has been previously discussed only by the author (Horst, 1965).

### 13.2 Summary of Results

We recall that  $m$  is the number of roots of the correlation matrix greater than unity and that for each data set this was the number of factors solved for. We recall also that the criterion  $\phi$  is the ratio of the sum of squares of the  $m$  largest roots of a matrix with specified loss and scaling parameters to the sum of squares of all of its roots. The iterations for  $\phi$  continued until the absolute value of the difference between two successive  $\phi$ 's was less than .0001 with an upper limit of 100 iterations. The  $\gamma$  criterion of simple structure is defined in Chapter 9. The tolerance limit for the simple structure criterion was approximately the same as for  $\phi$ . The actual function used for the tolerance limit is discussed in Chapter 12, Section 2. The iteration limit was 60. The limit on the number of sets of iterations was 20.

Table 1 summarizes results for the twelve data sets.

The first column gives simply the arbitrary serial numbers of the data sets.

The second column gives abbreviated identification of the data sets. The third column gives the number of factors solved for. The next column headed "Crit." (Criterion) gives for each data set first the approximation criterion  $\phi$  and below it the simple structure criterion  $\gamma$ . To the right of each  $\phi$  are respectively the number of iterations and the actual  $\phi$  value for the six combinations of loss and scaling parameters. The first six columns following the column of criterion symbols give the data for  $P_W = 1$  and the next six columns for  $P_W = 0$ . Within these sets of six columns, the first pair of columns gives the data for  $p = 0$  and the next two for  $p = .5$  and  $p = 1$  respectively.

### 13.3 Rankings by Simple Structure Criterion

Perhaps one of the most important questions to be answered is which of the six methods of analysis is the best as judged from the analysis of the twelve sets of data. One overall standard might be based on the simple structure criterion  $\gamma$ . Table 2 gives for each of the first nine data sets the rank order of the  $\gamma$  value for each of the six combinations of loss and scaling parameters. The first three columns of rankings are for  $P_W = 1$  and the last 3 for  $P_W = 0$ . Within each set of three columns, the first gives the ranking for  $p = 0$ , and the next two for  $p = .5$  and  $p = 1$ . Rankings for only the first nine of the twelve data sets are given, since the  $\gamma$ 's for data set 10 are all equal and no  $\gamma$ 's are given for data sets 11 and 12.

The  $\Sigma$  row following the row for data set 9 gives the sum of the rankings for each pair of loss and scaling parameters. The next row gives the rank order of the sum of the rankings of these sums. The next row H gives the number of data sets for which  $\gamma$  had the highest ranking, and the last row L gives the number of data sets for which  $\gamma$  had the lowest rank.

It is clear from the last three rows of this table that the best method according to the simple structure criterion is ESCOV for  $p = 0$  and  $P_W = 1$ . This is the

alpha factor analysis model of Kaiser and Caffrey (1965). The poorest method is REVAC for  $p = 0$  and  $P_W = 0$ . This is the model discussed by Anderson and Rubin (1956). In view of the problems encountered in this model, it is not surprising that it is poorest according to the simple structure criterion.

For these nine data sets, RECOV with  $p = 0$ ,  $P_W = 1$  is second poorest according to rankings of the simple structure criterion. This is the method closely related to the maximum likelihood and canonical models of Lawley (1940) and Rao (1955) respectively.

The methods in second and third place respectively are TOCOV with  $p = .5$  and  $P_W = 1$ , and TOVAC with  $p = .5$  and  $P_W = 0$ . These correspond respectively to the minres model of Harmon (1967) and to the classical principal components model.

Obviously, of course, the procedure we have used for the comparative evaluation of the six models is crude and is based on a very limited number of data sets. Furthermore, the criterion for the number of factors is arbitrary and other criteria may yield different results. In any case, it is quite possible that for any particular data set one may wish to determine the loss and scaling parameters  $p$  and  $P_W$  so as to maximize the criterion  $\Psi$ . Such a procedure need not limit the value of these parameters to those used in this analysis.

#### 13.4 Ranks by Number of Iterations Required

A further procedure for a relative evaluation of the six models may be based on the number of iterations required for  $\phi$  to stabilize to the specified tolerance limit. Table 3 provides an analysis similar to that of Table 2. Here, however, the rankings are for the number of iterations required for  $\phi$  given in Table 1 and all 12 data sets are included. In this ranking we exclude the 2-2 column since this is the principal axis method and except for peculiarities of the computer program, no iterations would be required. In any case, according to the last three rows of Table 3, ESVAC with  $p = 1$ ,  $P_W = 0$  is best and ESCOV with  $p = 1$ ,  $P_W = 1$  is

second best. This latter is the model which came out best in Table 2 based on the simple structure criterion  $\Psi$ . The poorest is RECOV with  $p = 0$  and  $P_w = 1$ , which came out second poorest in Table 2.

Again it is obvious that the rating procedure is crude and based on what some may regard as a questionable criterion for the number of factors. However, a cursory examination of Table 3 shows that in general the number of iterations required for  $\phi$  to stabilize tends to be substantially greater for RECOV than for the other models. It is believed that this tendency would persist even with other defensible criteria for the number of factors.

In view of the marked increase in iterations required for this model over those required for other models and at least some persuasive indications of poor simple structure potentiality, one might question whether this model is to be generally recommended. Since it is closely related to the maximum likelihood and canonical models, one might also question whether the great interest and effort accorded these models in the past is completely justified.

### 13.5 The Simple Structure Factors

A more detailed examination of the simple structure factor matrices given at the right of the tables in Chapter 12 for data sets 1 through 10 might be of interest. No simple structure matrices were calculated for data sets 11 and 12 since only one factor was obtained. The largest element in each row is underlined for all the simple structure matrices in data sets 1 through 10. For each data set there are six of these, one for each combination of the loss and scaling parameters. (See Chapter 12 for detailed description.) For the first nine data sets these simple structure matrices may be compared with those obtained by other investigators referred to in Chapter 11. Brief comments on the data sets might be of interest as follows:

1. Primary Mental Abilities. For all six models, the simple structure matrices are sharp and agree well with the results of Thurstone and Thurstone (1941).

2. Twenty-four psychological tests. According to the  $\Psi$  criterion, ESCOV with  $p = 1$ ,  $P_w = 1$  gives the best simple structure. This model corresponds to Kaiser's (1965) alpha model. A number of simple structure solutions are given by Harmon (1967) for this data set. However, his various solutions involve only four factors whereas ours has five. But all of his simple structure solutions give results easily recognized as similar. Our simple structure for ESCOV gives results similar to his for three of the factors. However, his solutions assign variables 14 through 19 essentially to a single factor, whereas ESCOV splits them into two factors, the first three going to one and the last three to another as follows:

- |                        |                     |
|------------------------|---------------------|
| 14. Word recognition   | 17. Object - number |
| 15. Number recognition | 18. Number - figure |
| 16. Figure recognition | 19. Figure - word   |

Without referring in detail to a description of the original tests, it is not surprising that recognition of various types of visual stimuli should have a factor in common, and ability to associate pairs involving two different types of stimuli should have another factor in common. It is quite possible, of course, that if Harmon had included a fifth factor in his analysis he also would have found the same factor differentiation between these two triplets of tests. It is interesting to note that our TOCOV solution which corresponds to Harmon's minres does not appear to give as sharp a simple structure for five factors as his does for the four factors on which he uses a direct oblimin solution.

3. Thirty-three variable speed study. Of the six models, ESCOV again gives the highest simple structure criterion for the data from Lord (1956). Lord solved for ten factors by a modification of the maximum likelihood method. Our criterion

for the number of factors yielded only six factors. Lord's rotations were carried out by subjective non-analytical procedures so that his simple structure matrix is not comparable to our ESCOV with  $P_w = 1$  and  $p = 1$ . However, referring to Section 11.3, the verbal, spatial, and number factors come out clearly as they do in Lord's analysis. In addition, a factor common to the two number speed tests (23) and (24) and Lord's reference tests for perceptual speed (25), (26), and (27) appears in our analysis. It is also interesting that in our analysis college grades tend to split, with English (28) going to the verbal factor, Engineering Drawing (30) to the spatial factor, and Foreign Language (29), Chemistry (31), and Mathematics (32) predominately to a factor that we might characterize as a facility with symbolic systems. A factor which Lord failed to find has only a single high loading on Conduct (33) and small positive loadings for the five grade variables (29) through (32). This may be a conformity factor considering the data are based on students in a military academy.

4. Thurstone's twenty-variable box problem. From this classical set of data there is little to choose among the simple structure factor matrices for the six models. As would be expected, three factors were obtained. With the exception of RECOV for  $p = 0$ ,  $P_w = 1$ , the  $\Psi$  values do not differ by more than .001. For RECOV the value is only .003 less than the highest value of .646 for TOCOV, ESCOV, TOVAC, and ESVAC. The simple structure is clear for all cases with the X, X, and Z dimension variables coming out with only a single large loading for a factor and the functions of these variables indicated in Section 11.4 having the loadings that would be expected.

5. Eight-variable body type measures. All six models for this data set, yielding only two factors, give very clear simple structure as has been found by other investigators.

6. Twelve-variable anthropometric measures. In an early analysis of these data, Thurstone (1946) found four simple structure factors by subjective graphical methods whereas our criterion gave only three factors. For all models except RECOV and REVAC whose loss parameter  $p$  is 0, the  $\Psi$  values are in the high .80's with ESCOV again being highest with  $\Psi = .880$ . The simple structures for all six models are clear cut. Thurstone's B and D factors have both loadings of .45 or more on the three variables stature (1), span (5), and hand length (11). In addition, his factor B has high loadings on sitting height (2), and his factor D has a high loading on hand breadth (12). Our first factor tends to collapse these two factors, while our second and third factors are easily recognized as Thurstone's factors C and A respectively.

7. Fifteen variables from Hemmerle. Hemmerle (1965) does not give the source of this matrix nor does he identify the variables. Since he was concerned primarily with a computational procedure for the maximum likelihood factor model, he did not attempt a simple structure transformation. He extracted eight factors but does not state his criterion. Our criterion got only five factors. These data were included in our study primarily because Jöreskog (1967) as well as Horst (1968b) had also worked extensively with them. Both of us had found the data to behave peculiarly and our earlier results for eight variables were markedly different for the maximum likelihood methods of Jöreskog and our corresponding RECOV model. Even in the present study it is the only data set that has its highest  $\Psi$  value (.596) for REVAC which, from a theoretical point of view, is the poorest of the six models. Since nothing is known about the identity of the variables, nothing of substantive interest can be said about their simple structure factor loadings.

8. Seventeen-variable data from Bechtold--Sample 1. For this data set our criterion yielded only five factors whereas Bechtold (1961) had deliberately attempted to represent six factors in his battery, as indicated in Section 11.8.

In general, his V, W, S, and N factors come out clearly in all six models. The memory factor for variables 1 and 2 fails to come out clearly in RECOV which has the highest  $\Psi$  value, although the reasoning factor R for variables 15, 16, and 17 comes out clearly in this model. The only other model for which R comes out clearly is the suspect REVAC which also fails on the M. It is quite probable that our criterion for number of factors was too low for this data set.

9. Seventeen-variable data from Bechtold--Sample 2. Since the tests in this data set were the same as for data set 8 and the sample was presumably comparable, the results should be substantially the same for the simple structure matrices. As in the previous set, the criterion yielded five factors. For all six models, the V, W, S, and N factors are clearly defined. As in the previous set, the M factor for variables 1 and 2 appears in all models except RECOV and REVAC but somewhat less clearly in TOCOV. The R factor for variables 15, 16, and 17 appears most clearly as a distinct factor in RECOV and TOCOV. Here again, it is highly probable that our criterion for number of factors was too restrictive.

10. Nine-variable synthetic data. The origin or source of this data set is described in detail in Chapter 11, Section 10. The correlation matrix was constructed so that the simple structure factor loading matrix would have three factors and, to three-decimal accuracy, this matrix would be as follows:

	I	II	III
1	.000	.383	.924
2	.000	.707	.707
3	.000	.924	.383
4	.383	.924	.000
5	.707	.707	.000
6	.924	.383	.000
7	.924	.000	.383
8	.707	.000	.707
9	.383	.000	.924



It can be shown that if one allocates three points on each of the arcs of a right spherical triangle as indicated in Chapter 11, Section 10, then to three decimal places the cosines of the angles of each of these nine points with each of the three vertices of the spherical triangle will be as shown in the matrix above. For some decades we have been trying to find an analytical procedure for recovering this matrix from the correlation matrix of these points. To our knowledge, none of the analytical methods previously available will accomplish this recovery. Reference to Chapter 12 for the results from this data set shows that the simple structure factor matrices for all six models differ at most from the above matrix by .001. For all six models, the sets of simple structure iterations went to the prespecified limit of twenty. For each model the number of iterations for the first set went to the prespecified limit of sixty. Thereafter, however, the number of iterations required for the successive sets diminished rapidly to two or three. It is quite probable that if no limit were placed on the number of sets of iterations, the original simple structure matrix could be recovered to any desired degree of accuracy.

### 13.6 Improper Solutions

A factor analysis model whose loss function involves only the residual covariances may yield communalities for some tests which exceed unity. Such a result is known as a Heywood case. These factor analysis models are our RECOV, TOCOV, and ESCOV which correspond respectively to Lawley's (1940) maximum likelihood, Harmon's (1967) minres, and Kaiser's (1965) alpha. It is of interest to note that for none of our data sets does our method of computation involving real data give communalities as high as the constrained values of .9995 for any of these three cases. This is not true for Harmon's (1967) minres on data set 2, where he obtains his maximum constrained communality of unity for variable 19.

When an unconstrained solution yields communalities greater than unity, this is sometimes called an improper solution. In the case of maximum likelihood solutions, Jöreskog (1967) observes that, "Experience verifies that improper solutions are found more often than is usually expected." Our RECOV computational algorithms have been applied to some of the data sets on which Jöreskog has applied his computational algorithms for the corresponding maximum likelihood method. In general, where we have used the same number of factors, neither of us has encountered an unconstrained improper solution. However, for the case of data set 7 from Hemmerle for eight factors, Jöreskog's (1967) procedure found it necessary to constrain variables 7 and 15 whereas our (Horst, 1968b) procedure up to 10,000 iterations found no improper communalities. A highly accelerated modification of our procedure did not require the constraining supplements in the computational procedures. Nevertheless, it is not improbable that for a completely adequate factor analysis system, the occurrence of improper communalities would signal either the use of inappropriate loss or scaling parameters, an inappropriate criterion for the number of factors, or some combination of these.

TABLE 1

NO.	DATA	No. of Var.	No. of Fact.	Crit.	$P_W = 1$						$P_W = 0$					
					1 - 1		1 - 2		1 - 3		2 - 1		2 - 2		2 - 3	
					RECOV - 0.0		TOCOV - 0.5		ESCOV - 1.0		REVAC - 0.0		TOVAC - 0.5		ESVAC - 1.0	
					Iter.	Crit.	Iter.	Crit.	Iter.	Crit.	Iter.	Crit.	Iter.	Crit.	Iter.	Crit.
1	...	9	3	✓	24	1.000	15	1.000	10	.999	16	1.000	4	.969	6	.965
	PMA				11	.959	14	.959	18	.958	15	.953	15	.963	20	.964
2	Psy Test	24	5	✓	53	.993	24	.989	9	.985	20	.999	4	.929	10	.923
					60	.592	60	.583	60	.661	60	.592	60	.629	60	.597
3	Lord	33	6	✓	20	.999	16	.998	10	.997	13	1.000	4	.969	10	.963
					53	.536	59	.620	44	.758	50	.646	60	.547	60	.511
4	Box	20	3	✓	31	1.000	8	1.000	8	1.000	15	1.000	3	1.000	3	1.000
					60	.643	60	.646	60	.646	37	.645	60	.646	60	.646
5	Body Types	8	2	✓	71	.999	14	.999	9	.999	11	1.000	4	.979	6	.974
					13	.984	14	.989	24	.990	18	.970	15	.989	24	.989
6	Anthro	12	3	✓	100	.995	15	.981	11	.975	9	1.000	4	.891	9	.869
					29	.791	12	.877	18	.880	31	.659	13	.874	21	.868
7	Hammerle	15	5	✓	92	.993	35	.981	12	.978	10	.999	4	.884	11	.879
					60	.569	24	.500	48	.496	47	.596	60	.469	60	.476
8	Bechtold - 1	17	5	✓	71	.998	16	.994	14	.994	13	1.000	4	.958	12	.956
					18	.875	25	.848	25	.840	45	.843	17	.846	19	.837
9	Bechtold - 2	17	5	✓	58	.998	31	.993	18	.989	14	1.000	4	.953	27	.946
					26	.861	37	.864	26	.906	42	.841	23	.904	25	.858
10	Synthetic	9	3	✓	8	.999	10	.999	10	.999	3	.999	3	.999	3	.999
					60+	.667	60+	.667	60+	.667	60+	.667	60+	.667	60+	.667
11	Davis	9	1	✓	13	.999	9	.998	7	.997	20	1.000	4	.925	8	.853
12	Thomson	5	1	✓	100	.981	11	.985	8	.987	5	1.000	4	.953	6	.935

TABLE 2

## SIMPLE STRUCTURE CRITERION RANKS

Data Sets	$P_W = 1$			$P_W = 0$		
	RECOV 1-1	TCCOV 1-2	ESCOV 1-3	REVAC 2-1	TOVAC 2-2	ESVAC 2-3
1	$3\frac{1}{2}$	$3\frac{1}{2}$	2	1	5	6
2	$2\frac{1}{2}$	1	6	$2\frac{1}{2}$	5	4
3	2	4	6	5	3	1
4	1	$4\frac{1}{2}$	$4\frac{1}{2}$	2	$4\frac{1}{2}$	$4\frac{1}{2}$
5	2	4	6	1	4	4
6	2	5	6	1	4	3
7	5	4	3	6	1	2
8	6	5	2	3	4	1
9	3	4	6	1	5	2
$\Sigma$	27	35	$41\frac{1}{2}$	$22\frac{1}{2}$	$35\frac{1}{2}$	$27\frac{1}{2}$
Rank	2	5	6	1	4	3
H	1	1	5	1	1	2
L	1	1	0	4	1	2

TABLE 3

RANKINGS BY NUMBER OF ITERATIONS REQUIRED

Data Sets	RECOV 1-1	TOCOV 1-2	ESCOV 1-3	REVAC 2-1	TOVAC 2-2	ESVAC 2-3
1	6	4	3	5	1	2
2	6	5	2	4	1	3
3	6	5	$2\frac{1}{2}$	4	1	$2\frac{1}{2}$
4	6	$3\frac{1}{2}$	$3\frac{1}{2}$	5	$1\frac{1}{2}$	$1\frac{1}{2}$
5	6	5	3	4	1	2
6	6	5	4	$2\frac{1}{2}$	1	$2\frac{1}{2}$
7	6	5	4	2	1	3
8	6	5	4	3	1	2
9	6	5	3	2	1	4
10	4	$5\frac{1}{2}$	$5\frac{1}{2}$	2	2	2
11	5	4	2	6	1	3
12	6	5		2	1	3
$\Sigma$	69	57	$40\frac{1}{2}$	$41\frac{1}{2}$	$13\frac{1}{2}$	$30\frac{1}{2}$
Rank	1	2	4	3	6	5
H	11	0	0	1	0	0
L	0	0	0	0	12	0

## CHAPTER 14

### COMPUTER PROGRAMS

The Fortran IV computer programs for carrying out the analyses of the previous chapters consist of a main program (MAIN) and overlay subroutine subprograms called by the main program. The overlay subroutines are called SYMI, JACS3, JACS, RARE, SIMP, and DUPLI.

#### 14.1 MAIN

The main program provides parameter values required for the computations, an outer loop for the data sets, an intermediate loop for the loss parameter, an inner loop for the scaling parameter, and a call to the output overlay subroutine DUPLI.

The parameters. It is standard practice to read in parameters from cards along with the data cards. Particularly is this true if the program deck is a binary deck. It is our opinion that binary program decks are essentially obsolete, especially with the rapid compilers currently available. It is usually desirable to have the source program immediately available with the output of a given computer run, together with all the program parameters and option codes that were used in the computer run. We have been repeatedly frustrated in attempting to assist laymen in the interpretation of their computer output by the fact that they used binary program decks and therefore could provide no information about the program parameters, option codes, and the computing algorithms utilized.

If a source program deck such as Fortran IV is used, it is possible to read in program parameters and option code cards as data and these cards can be varied to suit the requirements of the investigator and his data. However, it may be convenient in research with various data analysis models to provide some of this information in program statements so that they may be readily found at the beginning of the program listing. Some of these values are given at the beginning of MAIN. A number of them are repeated with different numerical values. The last time the

replacement statement appears for a parameter variable is of course the value it takes in the program. It has been found convenient for research purposes to provide a number of values which may be changed merely by changing the position of the statement.

The parameters area as follows:

P	tolerance limit
LIB	beginning indexing parameter for loss parameter
LIE	ending indexing parameter for loss parameter
LB	beginning indexing parameter for scaling parameter
LE	ending indexing parameter for scaling parameter
NL	iteration limit for simple structure iterations
NF1	ending indexing parameter for row scaling option of factor matrix
KKL	iteration limit for principal axis solution
EE	Tolerance limit for specificity variance

The outer loop. This is the loop with the index LLL and the indexing parameters 1, NP. This loop controls the number of data sets processed in a given run. The data for each set consists of the number of variables in the set, the format of the correlation matrix, and the correlation matrix itself. The loop calls the subroutine SYMI which provides initial estimates of the residual variances and JACS3 which determines the number of factors.

The intermediate loop. This is the loop with the index LLI and indexing parameters LIB, LIE. It calculates the loss parameter  $P_W$ . In this program the calculation of only  $P_W = 1$  and  $P_W = 0$  are provided for but any desired intermediate values could be provided with slight modification.

The inner loop. This loop has the index LL and the indexing parameters LB, LE. It writes the parameters LLL, N, LI, LLI, LL, and NFI on scratch tape. It calculates functions of the scaling parameter p which are used in the calculation of the scaling matrix. In this program only three loss function parameters are provided for. These are  $p = 0$ ,  $p = .5$ , and  $p = 1$ . However, as in the case of the loss parameter, any desired intermediate values could be provided with slight modification.

This loop also calls JACS which calculates a first approximation to the basic structure factor matrix and RARE which calculates iteratively the simple descaled matrix from the basic structure factor loading matrix. If, as is usually the case, the number of factors exceeds 1, this loop also calls SIMP which calculates the simple structure factor loading matrix.

DUPLI. This subroutine is outside the outer loop of MAIN. It reads from scratch tape the data that is to be printed and writes it on BCD tape in the format in which the data in Chapter 12 are given.

#### 14.2 SYMI

This overlay subroutine reads in the data, calculates the inverse of the correlation matrix, and then calculates the first approximation to the residual variances.

Data input. A single card giving the number of variables N is read with format (I4). The program has been dimensioned for up to 80 variables. It could probably be extended to 85 and, with some rewriting, to 90. An A-format card giving the format of the correlation matrix is read. The correlation matrix is read. As the



program is written, each row of the correlation matrix must begin on a new card. The program assumes that at least the infra-diagonal elements are given. The supra-diagonal elements must either be given or treated as zero. In either case the program then writes the supra-diagonal elements and enters unity in the diagonals. Then the correlation matrix is stored on scratch tape.

Matrix inversion. The program calls a regular subroutine SYMIN to invert the correlation matrix. If SYMIN finds that the correlation matrix is not basic or positive definite, it returns control to SYMI, the overlay subroutine, which shrinks the offdiagonal elements by a factor of .9. This factor is arbitrary. It can be shown that if the offdiagonal elements on any correlation matrix are multiplied by a positive value less than unity, the resulting matrix will be basic and hence have a regular inverse. SYMIN is again called to invert the modified correlation matrix.

Residual variance approximation. The reciprocals of the diagonal elements of the inverse of the correlation matrix are calculated. These provide the first approximation to the residual variances. They are written on scratch tape.

#### 14.3 JACS3

This subroutine reads the correlation matrix from scratch tape on which it was written by SYMI. It calculates, by an adaptation of the Jacobi method, all the roots of the correlation matrix in order of magnitude which are greater than unity. It transmits the number of these roots to common core storage as the number of factors.

#### 14.4 JACS

This overlay subroutine reads the necessary data from scratch tape. It then calculates the first approximation to the modified correlation matrix with specified loss and scaling parameters, and the first approximation to the basic structure matrix.

The data. The correlation matrix is read from scratch tape on which it was written. Without rewinding, the first approximation to the residual variance vector is read from the same tape.

The scaling matrix. The communality variance is calculated. The scaling matrix is calculated as a function of the communality and residual variance vectors and the scaling parameter.

The modified correlation matrix. The correlation matrix is scaled by the scaling matrix. The diagonals of the resulting matrix are adjusted according to the current loss parameter.

The basic structure matrix. Subroutine JACSIM is called. This subroutine calculates the first m principal component or basic structure vectors of the modified correlation matrix where m is the number of roots greater than unity found in Section 14.3. This is the first approximation to the principal axis matrix for a specified loss and scaling parameters. The principal axis matrix is written on scratch tape.

#### 14.5 RARE

This overlay subroutine calculates the descaled principal axis matrix for current loss and scaling parameters. It reads the necessary data from scratch tape. It then calculates a first approximation to a descaled principal axis matrix and a second approximation to the scaling matrix. It calculates iteratively the descaled principal axis matrix for predetermined loss and scaling parameters. It writes output data on scratch tape. Next it effects row sign reversals if needed. Finally, it transfers data to scratch tape.

Input data. The correlation matrix is read from scratch tape. The first approximation to the principal axis factor matrix is read from another scratch tape.

First descaled principal axis matrix. The descaling matrix for the first approximation to the factor loading matrix is calculated. The first approximation to the descaled factor loading matrix is calculated.

The scaling matrix. Second approximations to the communality and residual variance vectors are calculated. A second approximation to the scaling vector is calculated from these two vectors and functions of the scaling parameter.

Successive descaled principal axis matrices. A loop with index KKK and indexing parameters 1, KKL is set up to call iteratively subroutine RARED. This subroutine calculates successive approximations to the descaled principal axis matrix for the loss and scaling parameters determined within the inner loop of MAIN. The computations are carried out by the algorithms indicated in Chapter 8. The subroutine includes a constraint to keep the residual variances positive. It also calculates the criterion  $\phi$  of Chapter 12 and the difference between two successive  $\phi$ 's as a convergence tolerance.

Output data. The final  $\phi$  value, the number of iterations taken, and the total time in seconds are written on the scratch tape which will subsequently be read back for output. The first m roots of the final modified correlation matrix are also written on this tape.

Sign reversals. The first element in each row of the final descaled principal axis matrix is checked for sign. Sign reversals by row are made where necessary.

Transfers of output data to tape. The final descaled principal axis matrix, together with the sign vector and the final communality and residual variance vectors, are written on the scratch tape for output data. The final descaled principal axis matrix is also written on another scratch tape to be read subsequently for further operations.

#### 14.6 SIMP

This overlay subroutine calculates the simple structure factor loading matrix. It provides parameter values and options of row scaling for the descaled principal axis matrix. It has a major outer and an inner iteration loop for calculating the simple structure matrix. It writes the output data on the output scratch tape.

Parameter values. The parameter LLE gives the limit on the number of sets of iterations. The parameter ML is used in calculating the F value in Eq. 9.7.

Row scaling option. The program normalizes the principal axis matrix by rows before beginning the simple structure iterations. The final simple structure matrix is denormalized before being written on output tape. This solution is given by using the parameter NFL = 1 in MAIN. This parameter serves as the end indexing parameter in SIMP for the DO index LLL. If in addition to this solution it is also desired to have a solution without first normalizing by rows, the parameter NFL = 2 is used in MAIN. The program does not provide for just the non-normalized solution but with slight modification it can be made to do so.

Outer iteration loop. The major outer iteration loop has the index LLL4 with indexing parameters LLE, LLE. This loop provides for successive sets of iterations where the exponent F decreases with each succeeding set. The value F is calculated as a function of the index LLL4 and the parameter ML. For each iteration set, this loop also writes on the output scratch tape the tolerance criterion, the simple structure criterion, the number of iterations, and the number of the iteration set. For each set of iterations, this loop determines whether any vector of simple structure factor loadings has less than m negative values. If so, no further set of iterations is calculated.

Inner iteration loop. This loop has the index LL with the indexing parameters L, NL. It calculates iteratively the transformation matrix and the simple structure factor matrix by means of the algorithms given in Eqs. 9.57 through 9.71. For each iteration it calls the subroutine SYML3 which calculates the inverse of a positive definite symmetric matrix. Within this loop also is calculated the criterion value. If two successive values are within the tolerance limit, the iterations are terminated.

The final simple structure matrix. After the successive sets of iterations are terminated, the program recognizes as the simple structure factor matrix the one calculated in the next to last set of iterations, unless only one set was calculated. In the latter case, the matrix calculated in the single set of iterations is recognized as the simple structure matrix. For the calculations beginning with a row normalization of the principal axis factor matrix, the final simple structure matrix is denormalized by rows. In either case, the final simple structure matrix is written on the output scratch tape. The vector of  $\Psi$  criterion values for each simple structure factor vector is also written on the output scratch tape.

#### 14.7 DUPLI

This overlay subroutine reads the data on the output scratch tape and writes it on BCD output tape according to the format of the data in Chapter 12. The subroutine has an outer loop with index LS and indexing parameters 1, NS so that NS copies of the output will be printed.

```

      MAIN
      DIMENSION JI(90)
      COMMON P,NL,N,NF,L,KI,KKL,KK2L,NA,E1,EF,KK3L,HH,KKK
      *,NC,FF1,FF2,LL1
      *,PD,QD,PW
      *,NF1
      *,TIM
      *,LI
      *,LLA,NP,LIR,LIE,LP,LF,JI
      P = .00001
      NP=7
      NP=8
      NP=5
      NP=3
      NP=1
      NP=4
      NP=2
      NP=12
      LR=3
      LIR=2
      LR=2
      LIE=2
      LR=1
      LIE=1
      LIR=1
      LIF=2
      LF=1
      LF=3
      NL=30
      NL=100
      NL=60
      NF1=2
      NF1=1
      KKL=10
      KKL=50
      KKL=100
      EF=.005
      EF=.0005
      REWIND 8
      L7=0
      WRITE(6,999)
999  FORMAT(1H1)
      DO 998 LLL=1,NP
      CALL SYM1
      CALL JACS2
      DO 992 LLI=LIR,LIF
      PW=2-LLI
      DO 991 LL=LR,LF
      WRITE(6,997)LLL,N,LI,LLI,LL,NF1
997  FORMAT(6F15)
      WRITE(6,997)
992  FORMAT(//)
      L7=L7+1
      WRITE(14)11,N,LI,LLI,LL,NF1
      DO 993 LL

```

```
PD=PD/2.  
QD=1.-PD  
PQ=1.-?.*PD*QD  
PD=PD/PQ  
QD=QD/PQ  
CALL JACS  
CALL RARE  
WRITE(6,993)  
993 FORMAT(///)  
IF(LLI-1)980,831,880  
890 CONTINUE  
CALL SIMP  
JI(L7)=LLA  
891 CONTINUE  
882 CONTINUE  
WRITE(6,995)  
888 CONTINUE  
REWIND 8  
WRITE(6,999)  
CALL DUPLI  
STOP  
END
```

SORIGIN        ALPHA  
\$IBFTC SYM111

```

SUBROUTINE SYM1
  DIMENSION R(80,80),Y(80,80),A(80)
  *,DE(80)
  *,FM(12)
  COMMON P,NL,N,NF,L,KL,KKL,KK2L,NA,E1,EE,KK3L,HH,KKK
  *,NC,FF1,FF2,LL1
  *,PD,QD,PW
  *,NF1
  *,TIM
  *,LI
  *,LLA,NP,L1B,L1F,LB,LE,JI
  REWIND 2
  REWIND 3
  REWIND 4
  READ(5,992)N
992 FORMAT(I4)
  READ(5,991)(FM(I),I=1,12)
991 FORMAT(12A6)
  DO 502 I=1,N
  READ(5,FM)(R(I,J),J=1,N)
502 CONTINUE
  DO 4 I=1,N
  DO 2 J=1,N
  R(I,J)=R(J,I)
  2 CONTINUE
  R(I,I)=1.
  4 CONTINUE
  DO 12 I=1,N
  WRITE(2)(R(I,J),J=1,N)
  12 CONTINUE
  IS=0
  CALL SYMIN(R,N,IS)
  IF(IS)60,64,60
  60 CONTINUE
  REWIND 2
  DO 61 I=1,N
  READ(2)(R(I,J),J=1,N)
  61 CONTINUE
  DO 63 I=1,N
  DO 62 J=1,N
  R(I,J)=R(I,J)*.9
  62 CONTINUE
  R(I,I)=1.
  63 CONTINUE
  CALL SYMIN(R,N,IS)
  64 CONTINUE
  DO 501 I=1,N
  D(I)=1./R(I,I)
  501 CONTINUE
  WRITE(2)(DE(I),I=1,N)
  REWIND 2
  RETURN
END

```



SIBFTC SYM11

```

SUBROUTINE SYMIN (S,N,IS)
DIMENSION S(80,1)
N1 = N-1
DO 04 I = 2,N
  I1 = I-1
  DO 04 J = 1,I1
04   S(I,J) = 0.0
    C = 1.0/SQRT(S(1,1))
    S(1,1) = 1.0
    DO 13 J=1,N
13   S(1,J) = S(1,J) * C
    DO 21 K=2,N
    DO 17 J=1,N
    K1 = K-1
    DO 17 I=1,K1
17   S(K,J) = S(K,J) - S(I,K) * S(I,J)
    IF(-S(K,K))60,61,61
    60 CONTINUE
    C = 1.0/SQRT(S(K,K))
    DO 191 I=1,K1
191  S(I,K) = 0.0
    S(K,K) = 1.0
    DO 21 J=1,N
21   S(K,J) = S(K,J) * C
    DO 30 J=2,N
    J1 = J-1
    DO 30 I=1,J1
    DO 30 K=J,N
30   S(I,J) = S(I,J) + S(K,I) * S(K,J)
    DO 35 J=1,N1
    S(J,J) = S(J,J) **2
    J2 = J+1
    DO 35 I=J2,N
35   S(J,J) = S(J,J) + S(I,J)**2
    S(N,N) = S(N,N)**2
    DO 42 I=1,N1
    I2 = I+1
    DO 42 J=I2,N
42   S(J,I) = S(I,J)
    GO TO 62
    61 CONTINUE
    IS=1
    62 CONTINUE
    RETURN
    END

```

SORIGIN        ALPHA  
\$IBFTC JACS2

```

SUBROUTINE JACS3
  DIMENSION R(160,80),D(80)
  COMMON P,NL,N,NF,L,KL,KKL,KK2L,NA,E1,EE,KK3L,HH,KKK
  *,NC,FF1,FF2,LL1
  *,PD,QD,PW
  *,NF1
  *,TIM
  *,LI
  *,LLA,NP,LIB,LIE,LB,LE,JI
  DO 53 I=1,N
    READ (2)(R(I,J),J=1,N)
53  CONTINUE
    REWIND 2
06   N1 = N+1
061  N11=N-1
07   N2 = N*2
08   DO 10 I=N1, N2
09   DO 10 J = 1,N
10   R(I,J) = 0.
11   DO 12 I=1,N
111  NI = N + I
12   R(NI,I) = 1.
      RIM=1.
      DO 36 I = 1,N11
        I1 = I+1
13   DO 35 L = 1,NL
        AB=0.
        DO 284 J=I1,N
          RIJ=ABS(R(I,J))
          AB=AMAX1(AB,RIJ)
          IF(P-RIJ)40,42,42
40   CONTINUE
          LR=1
          DR=R(I,I)-R(J,J)
          DRR=DR**2
          AK=SQRT(DRR/(DRR+4.*R(I,J)**2))
          SD=SIGN(1.,DR)
          A=SQRT((1.+SD*AK)/2.)
27   B = SQRT (1.-A**2)
221  C = SIGN (1.,R(I,J))
          AC=A*C
          BC=B*C
23   DO 252 K = 1,N2
            U = R(K,I)*AC + R(K,J)*B
            R(K,J) = -R(K,I)*BC + R(K,J)*A
252  R(K,I) = U
            R(I,I)=R(I,I)*AC+R(J,I)*B
            R(J,J)=-R(I,J)*BC+R(J,J)*A
            R(I,J)=0.
            R(J,I)=0.
            DO 283 K=1,N
              R(I,K)=R(K,I)
              R(J,K)=R(K,J)
283  CONTINUE
42  CONTINUE

```

```
284 CONTINUE
    IF(P-AB)44,43,43
44 CONTINUE
35 CONTINUE
43 CONTINUE
    IF(RIM-R(I,I))45,46,46
45 CONTINUE
    LI=I
36 CONTINUE
46 CONTINUE
    DO 332 I=1,LI
332 D(I) = R(I,I)
    RETURN
END
```

\$ORIGIN        ALPHA  
\$IBFIC JACS1

```

SUBROUTINE JACS
  DIMENSION R(160,80),D(80),A(80)
  *,DE(80),DA(80)
  COMMON P,NL,N,NF,L,KL,KKL,KK2L,NA,E1,FE,KK3L,HH,KKK
  *,NC,FF1,FF2,LL1
  *,PD,QD,PW
  *,NF1
  *,TIM
  *,LI
  *,LLA,NP,LIB,LIF,LB,LF,JI
  DO 504 I=1,N
  READ(2)(R(I,J),J=1,N)
504 CONTINUE
  READ(2)(DE(I),I=1,N)
  REWIND 2
  PK=0.
  DO 61 I=1,N
  DA(I)=1.-DE(I)
  A(I)=1./SQRT(PD*DE(I)+QD*DA(I))
  PK=AMAX1(PK,PW*DE(I)*A(I)**2)
 61 CONTINUE
  DO 63 I=1,N
  DO 62 J=1,N
  R(I,J)=A(I)*R(I,J)*A(J)
 62 CONTINUE
  R(I,I)=(1.-PW*DE(I))*A(I)**2+PK
 63 CONTINUE
  CALL JACSIM(R,D,P,N,NL,LI)
  L=LI
  DO 507 J=1,L
  D(J)=SQRT(D(J)-PK)
  DO 505 I=1,N
  IN=I+N
  R(I,J)=R(IN,J)*D(J)
 505 CONTINUE
  WRITE(4)(R(I,J),I=1,N )
 507 CONTINUE
  REWIND 4
  RETURN
END

```

## \$IBFTC JACSI1

```

SUBROUTINE JACSIM (R,D,P,N,NL,LI)
DIMENSION R(160,80),D(80)
06  N1 = N+1
061 N11=N-1
07  N2 = N*2
08  DO 10 I=N1, N2
09  DO 10 J = 1,N
10  R(I,J) = 0.
11  DO 12 I=1,N
111 NI = N + I
12  R(NI,I) = 1.
   DO 36 I = 1,N11
   I1 = I+1
13  DO 35 L = 1,NL
   AB=0.
   DO 284 J=I1,N
   RIJ=ABS(R(I,J))
   AB=AMAX1(AB,RIJ)
   IF(P-RIJ)40,42,42
40  CONTINUE
   LR=1
   DR=R(I,I)-R(J,J)
   DRR=DR**2
   AK=SQRT(DRR/(DRR+4.*R(I,J)**2))
   SD=SIGN(1.,DR)
   A=SQRT((1.+SD*AK)/2.)
22  B = SQRT (1.-A**2)
221 C = SIGN (1.,R(I,J))
   AC=A*C
   BC=B*C
23  DO 252 K = 1,N2
   U = R(K,I)*AC + R(K,J)*B
   R(K,J) = -R(K,I)*BC + R(K,J)*A
252 R(K,I) = U
   R(I,I)=R(I,I)*AC+R(J,I)*B
   R(J,J)=-R(I,J)*BC+R(J,J)*A
   R(I,J)=0.
   R(J,I)=0.
   DO 283 K=1,N
   R(I,K)=R(K,I)
   R(J,K)=R(K,J)
283 CONTINUE
42 CONTINUE
284 CONTINUE
   IF(P-AB)44,43,43
44 CONTINUE
35 CONTINUE
43 CONTINUE
   IF(LI-I)45,46,45
45 CONTINUE
36 CONTINUE
46 CONTINUE
   DO 332 I=1,LI
332 D(I) = R(I,I)
   RETURN
END

```

SORIGIN        ALPHA  
\$IBFTC RARE1

```

SUBROUTINE RARE
  DIMENSION R(80,80),AM(80,30),UM(80,30),WM(110,30)
  *,D(80),U(80)
  *,A(150)
  *,AA(150)
  *,DE(80),DA(80)
  *,UE(80)
  COMMON P,NL,N,NF,L,KL,KKL,KK2L,NA,E1,EF,KK3L,HH,KKK
  *,NC,FF1,FF2,LL1
  *,PD,QD,PW
  *,NF1
  *,TIM
  *,LI
  *,LLA,NP,LIB,LIE,LB,LE,JI
  TIM1=TIME(2)
  DO 701 I=1,N
    READ(2)(R(I,J),J=1,N)
701  CONTINUE
    REWIND 2
    DO 702 J=1,L
      READ(4)(AM(I,J),I=1,N)
702  CONTINUE
      REWIND 4
      DO 63 I=1,N
        DE(I)=1.
        DO 61 J=1,L
          DE(I)=DE(I)+AM(I,J)**2
61  CONTINUE
          D(I)=1./SQRT(DE(I))
          DO 62 J=1,L
            AM(I,J)=D(I)*AM(I,J)
62  CONTINUE
            DA(I)=(DE(I)-1.)/DE(I)
            DE(I)=1.-DA(I)
            D(I)=PD*DE(I)+QD*DA(I)
63  CONTINUE
            LN=L+N
            DO 347 KKK=1,KKL
              CALL RAREDIR,UM,AM,WM,D,N,L,LN,U,KKK,KKL,AL
              *,EF
              *,UF
              *,C2
              *,ALM
              *,PD,PW,QD,DA,DE)
              AKK=KKK
              AA(KKK)=ALM
              A(KKK)=C2
              IF(P-AL)347,347,3471
347  CONTINUE
3471 CONTINUE
              WRITE(6,908)(AA(I),I=1,KKK)
              WRITE(6,992)
992  FORMAT(//)
              WRITE(6,908)(A(I),I=1,KKK)
908  FORMAT(10F8.4)

```

```
TIM2=TIME(2)
TIM=TIM2-TIM1
WRITE(8)C2,AKK,TIM
WRITE(8)(WM(I,I),I=1,LI)
SI=0.
DO 30 I=1,N
D(I)=SIGN(1.,AM(I,1))
SI=SI+D(I)
30 CONTINUE
SI=SIGN(1.,SI)
DO 34 I=1,N
D(I)=D(I)*SI
DO 32 J=1,L
AM(I,J)=AM(I,J)*SI
32 CONTINUE
WRITE(8)D(I),DA(I),DE(I),(AM(I,J),J=1,LI)
34 CONTINUE
DO 1111 J=1,L
WRITE(3)(AM(I,J),I=1,N)
1111 CONTINUE
REWIND 3
RETURN
END
```

\$IRFTC RARED1

```

      SUBROUTINE RARED(P,UM,AM,WM,D,N,L,LN,U,KKK,KKL,AL
      *,EE
      *,UE
      *,C2
      *,ALM
      *,PD,PW,QD,CA,DE)
      DIMENSION R(80,1),UM(80,1),AM(80,1),WM(110,1),D(80),U(80)
      *,DE(80),DA(80)
      *,UE(80)
      DO 315 I=1,N
      R(I,1)=1.-PW*DE(I)
      DO 315 J=1,L
315  UM(I,J) = (AM(I,J) / D(I))
      DO 321 I=1,N
      IL = I + L
      DO 321 J=1,L
      WM(IL,J)=0.
      DO 321 K=1,N
321  WM(IL,J) = WM(IL,J) + R(I,K) * UM(K,J)
      DO 328 I=1,L
      DO 328 J=1,L
      WM(I,J) = 0.0
      DO 327 K=1,N
      KL = K + L
327  WM(I,J) = WM(I,J) + WM(KL,I) * UM(K,J)
328  WM(J,I)=WM(I,J)
      C1=0.
      DO 2 I=1,L
      C1=C1+WM(I,I)
      2  CONTINUE
      DO 336 K=1,L
      S = 1./SQRT(WM(K,K))
      DO 331 I=K,LN
331  WM(I,K) = WM(I,K) * S
      K1 = K + 1
      IF (L-K) 343, 343, 334
334  DO 336 J=K1,L
      DO 336 I=J,LN
336  WM(I,J) = WM(I,J) - WM(I,K) * WM(J,K)
343  CONTINUE
      DO 705 I=1,N
      DE(I)=1.
      IL=I+L
      DO 701 J=1,L
      DE(I)=DE(I)-WM(IL,J)**2
701  CONTINUE
      IF(EE-DE(I))704,702,702
702  CONTINUE
      DD=SQRT((1.-EE)/(1.-DE(I)))
      DO 703 J=1,L
      WM(IL,J)=WM(IL,J)*DD
703  CONTINUE
      DE(I)=EE
704  CONTINUE
      DA(I)=1.-DE(I)

```



```

      UE(I)=DE(I)
      D(I)=PD*DE(I)+QD*DA(I)
      U(I)=D(I)
705  CONTINUE
      AL=C2
      HH=0.
      H1=((1.-PW*DE(N))/D(N))**2
      N1=N-1
      DO 11 I=1,N1
      H1=H1+((1.-PW*DE(I))/D(I))**2
      I1=I+1
      DO 11 J=I1,N
      H=R(I,J)
11  HH=HH+H**2/(D(I)*D(J))
      HH=H1+2.*HH
      C2=C1/HH
      ALM=0.
      DO 711 I=1,N
      IL=I+L
      DO 710 J=1,L
      ALM=AMAX1(ALM,ABS(AM(I,J)-WM(IL,J)))
      AM(I,J)=WM(IL,J)
710  CONTINUE
711  CONTINUE
      AL=ABS(AL-C2)
      RETURN
      END

```

\$ORIGIN        ALPHA  
\$IBFTC SIMP1

```

SUBROUTINE SIMP
  DIMENSION A(80,30),A1(80,30),E(30,30)
  *,B(80,30),S(30,30),H(30,30),D(80)
  *,D1(80)
  *,D2(80)
  *,DD(80)
  *,RC(80,30)
  *,G(80,30)
  *,DL(30)
  *,CB(30)
  *,DF(30)
  COMMON P,NL,N,NF,L,KL,KKL,KK2L,NA,E1,EE,KK3L,HH,KKK
  *,NC,FF1,FF2,LL1
  *,PD,QD,PW
  *,NF1
  *,TIM
  *,LI
  *,LLA,NP,LIB,LIE,LB,LE,JI
  M=L
  FM=M
  FN=N
  LLE=2
  LLE=4
  LLE=3
  LLE=NL
  LLE=1
  LLE=9
  LLE=1
  LLE=10
  LLE = 5
  LLE=20
  ML = 4
  ML=3
  ML=1
  ML=2
  NN=0
  DO 41 LLL=1,NF1
  DO 2 J=1,M
  *EAD(3)(A(I,J),I=1,N)
2 CONTINUE
  REWIND 3
  DO 52 I=1,N
  DD(I)=0.
  DO 51 J=1,M
  DD(I)=DD(I)+A(I,J)**2
51 CONTINUE
  DD(I)=SQRT(DD(I))
  DO 52 J=1,M
  IF(NN)511,510,511
510 CONTINUE
  A(I,J)= A(I,J)/DD(I)
511 CONTINUE
  B(I,J)=A(I,J)
52 CONTINUE
53 CONTINUE

```

```

F1=0.
DO 42 LLL4=LLR, LLE
ALL=LLL4
FM=LLL4*ML
F=2.*FM/(2.*FM-1.)
FP1=F+1.
FFF=1./(F-3.)
DO 82 J=1, M
DF(J)=D(J)
DO 82 I=1, N
G(I, J)=B(I, J)
92 CONTINUE
F1=0.
DO 20 LL=1, NL
AL=LL
DO 4 J=1, M
D1(J)=0.
D(J)=0.
DO 3 I=1, N
D1(J)=D1(J)+A(I, J)**4
D(J)=D(J)+ABS(B(I, J))**FP1
3 CONTINUE
D(J)=(D1(J)/D(J))**FFF
D1(J)=D(J)
DO 4 I=1, N
B(I, J)=h(I, J)*D(J)
4 CONTINUE
DO 7 I=1, N
D(I)=0.
DO 6 J=1, M
D(I)=D(I)+B(I, J)**2
6 CONTINUE
DO 7 J=1, M
A(I, J)=D(I)*A(I, J)
AC(I, J)=ABS(B(I, J))**F
7 CONTINUE
DO 9 I=1, M
DO 9 J=1, M
S(I, J)=0.
F(I, J)=0.
DO 8 K=1, N
S(I, J)=S(I, J)+A(K, I)*A(K, J)
F(I, J)=F(I, J)+A(K, I)*AC(K, J)
8 CONTINUE
9 CONTINUE
CALL SYM13(5, M)
DO 10 I=1, M
DO 10 J=1, M
H(I, J)=0.
DO 10 K=1, M
H(I, J)=H(I, J)+S(I, K)*F(K, J)
10 CONTINUE
C=0.
C1=0.
DO 240 J=1, M
D(J)=0.

```

```

      DO 23 I=1,M
      D(J)=D(J)+H(I,J)**2
23  CONTINUE
      D(J)=SQRT(D(J))
      C=C+D(J)
      DO 24 I=1,M
      H(I,J)=H(I,J)/D(J)
24  CONTINUE
      G(J)=D(J)/C1(J)
      C1=C1+D(J)
240 CONTINUE
      C=C/FMM
      C1=C1/FMM
      DO 26 J=1,M
      DO 25 I=1,N
      B(I,J)=0.
      DO 251 K=1,M
      B(I,J)=B(I,J)+A(I,K)*H(K,J)
251 CONTINUE
25  CONTINUE
26  CONTINUE
      F2=F1
      F1=C
      IF(P-(ABS(F1-F2))*2.1522,528,528)
522 CONTINUE
27  CONTINUE
524 CONTINUE
      LLA=LL14
      WRITE(810,C1,AL,ALL)
      F2=F1
      F1=C
      IF(P-(ABS(F1-F2))*2.1700,708,708)
700 CONTINUE
      AM=7.*FMM
      DO 404 J=1,M
      DB(J)=0.
      DO 403 I=1,N
      IF(R(I,J)+P)401,402,402
401 CONTINUE
      DB(J)=DB(J)+1.
402 CONTINUE
403 CONTINUE
      AM=AM/H(AM,DB(J))
404 CONTINUE
      IF(FMM-AM)266,266,40
266 CONTINUE
42 CONTINUE
40 CONTINUE
      IF(ALL-1.1702,706,702)
702 CONTINUE
      DO 704 J=1,M
      G(J)=DF(J)
      DO 704 I=1,N
      H(I,J)=G(I,J)
704 CONTINUE
706 CONTINUE

```

```
708 CONTINUE
    IF(NN)321,320,321
320 CONTINUE
    DO 34 I =1,N
    DO 34 J =1,M
        A(I,J)=DD(I)*B(I,J)
34 CONTINUE
321 CONTINUE
    DO 36 I =1,N
        WRITE(8)(B(I,J),J=1,LI)
36 CONTINUE
        WRITE(8)(D(J),J=1,LI)
    NN=1
41 CONTINUE
    RETURN
    END
```

SIBFTC SYM12

```

      SUBROUTINE SYM13 (S,N)
      DIMENSION S(30,30)
38    N1 = N-1
01    DO 04 I = 2,N
02      I1 = I-1
03      DO 04 J = 1,I1
04        S(I,J) = 0.
      10 C = 1./SQRT(S(1,1))
11      S(1,1) = 1.
12      DO 13 J=1,N
13        S(1,J) = S(1,J) * C
14      DO 21 K=2,N
15        DO 17 J=1,N
151       K1 = K-1
16        DO 17 I=1,K1
17          S(K,J) = S(K,J) - S(I,K) * S(I,J)
      19 C = 1./SQRT(S(K,K))
19      DO 191 I=1,K1
191       S(I,K) = 0.
192       S(K,K) = 1.
20      DO 21 J=1,N
21        S(K,J) = S(K,J) * C
25      DO 30 J=2,N
26        J1 = J-1
27        DO 30 I=1,J1
29        DO 30 K=J,N
30          S(I,J) = S(I,J) + S(K,I) * S(K,J)
31        DO 35 J=1,N1
32          S(J,J) = S(J,J) **2
33          J2 = J+1
34          DO 35 I=J2,N
35            S(J,J) = S(J,J) + S(I,J)**2
351       S(N,N) = S(N,N)**2
39      DO 42 I=1,N1
40        I2 = I+1
41        DO 42 J=I2,N
42          S(J,I) = S(I,J)
      RETURN
      END

```

SORIGIN      ALPHA  
SIBFTC DUPL II

```

SUBROUTINE DUPL I
  DIMENSION A(80,30),D(80),DA(80),DE(80),JI(80)
  *,D1(80)
  COMMON P,NL,N,NF,L,KL,KKL,KK2L,NA,EI,EE,KK3L,HH,KKK
  *,NC,FF1,FF2,LL1
  *,PD,QD,PW
  *,NF1
  *,TIM
  *,LI
  *,LLA,NP,L1B,L1E,LB,LE,JI
  NS=6
  NS=4
  NS=1
  NS=2
  DO 26 LS=1,NS
  L7=0
  DO 24 LLL=1,NP
  WRITE(6,999)
999 FORMAT(1H1)
  DO 22 LLI=L1B,L1E
  DO 20 LL=LB,LE
  WRITE(6,996)
996 FORMAT(///)
  L7=L7+1
  READ(8)LLL,N,LI,LLI,LL,NF1
  READ(8)C2,AKK,TIM
  READ(8)(D(I),I=1,LI)
  WRITE(6,901)LLL,N,LI,LLI,LL,NF1,C2,AKK,TIM,(D(I),I=1,LI)
901 FORMAT(6I4,3X,3F7.3,3X,7F10.3)
  WRITE(6,997)
997 FORMAT(1H1)
  DO 2 I=1,N
  READ(8)D1(I),DA(I),DE(I),(A(I,J),J=1,LI)
  2 CONTINUE
  IF(LI-1)8,15,8
  8 CONTINUE
  LLA=JI(L7)
  DO 4 I=1,LLA
  READ(8)C,C1,AL,ALL
  WRITE(6,902)C,C1,AL,ALL
902 FORMAT(5F7.3)
  4 CONTINUE
  WRITE(6,997)
  DO 14 I=1,N
  READ(8)(D(J),J=1,LI)
  WRITE(6,905)(D(J),J=1,LI)
905 FORMAT(78X,7F7.3)
  WRITE(6,906)I,D1(I),DA(I),DE(I),(A(I,J),J=1,LI)
906 FORMAT(1H+,I3,1X,1F4.0,1X,2F7.3,2X,7F7.3)
  14 CONTINUE
  WRITE(6,997)
  READ(8)(D(J),J=1,LI)
  WRITE(6,904)(D(J),J=1,LI)
904 FORMAT(2X,7F7.3)
  GO TO 18

```

```
15 CONTINUE
   DO 16 I=1,N
   WRITE(6,903)I,D1(I),DA(I),DE(I),(A(I,J),J=1,1)
903 FORMAT(13,1X,1F4.0,1X,2F7.3,2X,14F7.3)
16 CONTINUE
18 CONTINUE
20 CONTINUE
22 CONTINUE
24 CONTINUE
   WRITE(6,999)
   PEWIND 8
26 CONTINUE
   RETURN
   END
```



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<p>This document is Part II of a two-part report. In Part I a generalized scale free factor analysis method with variable loss function was developed, and new rationales for simple structure transformation and computation of factor scores were developed. In this report, the techniques are applied to twelve different data sets, including some of the classical sets reported in the literature by other investigators. Computer program listings are included.</p>			

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